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TRANSACTIONS
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OF
ELECTRICAL ENGINEERS.

REGULAR MEETING, NEW YORK, SEPTEMBER 27TH, 1892.

On the Law of Hysteresis, Part II.

BY CHARLES PROTEUS STEINMETZ.

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SUPERVISION OF THE EDITING COMMITTEE.
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AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

New York City, September 27th, 1892.

The sixty-ninth meeting of the Institute was held this date. The meeting was called to order by President Sprague.

THE PRESIDENT:—We meet to-night for the first time after the summer vacation. The paper that is going to be presented to you is one of great interest. It embodies the results of investigations which have been made by one of the ablest mathematicians of this Institute, carried on for months both day and night with resources which were practically unlimited in their experimental character, and they have been embodied in a paper which I think may fairly be said to be one of the most important ever presented here.

Owing to the pressure of private duties which has borne heavily on me for some time, I shall not be able to preside at this meeting and I will request Mr. Hammer to take my place. If there is any new business to present, the Secretary will do that in connection with the announcement of the election of new members.

THE SECRETARY:—At the meeting of the Council held this afternoon, the following associate members were elected:

Name.	Address.	Endorsed by
ALBRIGHT, H. FLEETWOOD,	Electrical Engineer, Western Electric Co., 227 So. Clinton St., Chicago, Ill.	G. M. Phelps. E. M. Barton. Chas. A. Brown.
ARMSTRONG, CHAS. G.	Electrical Expert and Electrical Architect, 1301 Auditorium Tower, Chicago, Ill.	F. J. Sprague. C. T. Hutchinson. Louis Duncan.
CALLENDER, ROMAINE	Electrician, Brantford Electrical Laboratory, Brantford, Canada.	T. J. Smith. F. Jarvis Patten. Ralph W. Pope.
CRANDALL, CHESTER D.	Assistant Treasurer, Western Electric Co., 227 South Clinton St., Chicago, Ill.	E. M. Barton. Geo. M. Phelps. Chas. A. Brown.
FISHER, GEORGE E.	General Manager, Commercial Electric Co., 55-57 Gratiot Ave., Detroit, Mich.	Elias E. Ries. Ralph W. Pope. Fred'k Reckenzaun.

FLESCH, CHARLES	Electrical Engineer, Melbourne, Australia.	Jos. Wetzler. T. C. Martin. Geo. W. Davenport.
JACKSON, J. P.	Assistant Professor of Electrical Engineering, Penn. State College, State College, Pa.	D. C. Jackson. Gilbert Wilkes. W. G. Whitmore.
KINSMAN, FRANK E.	Electrical Engineer, Plainfield, N. J.	Geo. A. Hamilton. Ralph W. Pope. H. C. Townsend.
MAGENIS, JAMES P.	Editor the *Adams Freeman*, Adams, Mass.	Frank J. Sprague. P. B. Delany. C. E. Dressler.
MACFADDEN, CARL K.	Chief Electric Light Inspector, Chicago & Northwestern Ry. Co., 22 Fifth Ave., Chicago, Ill.	R. W. Pope. Fred DeLand. A. H. Bauer.
MCBRIDE, JAMES	Superintendent, N. Y. & Boston Dye Wood Co., 146 Kent St., Brooklyn, N. Y.	W. A. Rosenbaum. J. A. Seely. Ralph W. Pope.
NOLL, AUGUSTUS	New York Insulated Wire Co., 15 Cortlandt St., New York City.	Jos. Wetzler. T. C. Martin. F. J. Sprague.
RAY, WILLIAM D.	Electrician of Local Line of Northern Pacific R. R. Co., at Chicago, 308 Home Ave., Oak Park, Ill.	D. C. Jackson. Fred. DeLand. Ralph W. Pope.
RODGERS, HOWARD S.	Electrical Engineer, Thomson-Houston Electric Co., 624 Western Ave., Lynn, Mass.	Franklin Sheble. Caryl D. Haskins. H. G. Reist.
ROSS, ROBERT A.	Engineer in charge of Engineering Dept., Edison General Electric Co., Petersborough, Ont.	John Langton. Wm. S. Andrews. Samuel Insull.
SMITH, FRANK STUART	Supt. of Carbon Dept., Westinghouse Electric & Mfg. Co., Pittsburg, Pa.	Chas. A. Terry. O. B. Shallenberger. Chas. F. Scott.

Total, 16.

Probably at one of the following meetings the Committee on Units and Standards, which has been pursuing its work for the last year or two will bring up a report for consideration by the Institute at large, in accordance with the action of the Council. We have a few proof copies of this report which I will be glad to have any of the members who are interested in this subject take with them in view of discussion at some future date.

THE PRESIDENT:—It is good for the Institute that we have at each returning meeting such a list of new members. I am glad to notice that the number of members, who either under the pressure of personal business or for other reasons, have found it necessary to drop out of the Institute are few.

The paper this evening will be by Mr. Charles P. Steinmetz. It is the second paper "On the Law of Hysteresis, and other Phenomena of the Magnetic Circuit." His work in the past has been most important in its character and this paper will fully support the reputation he has already earned.

The following paper was then read by the author.

A paper read at the sixty-ninth meeting of the American Institute of Electrical Engineers, New York, September 27th, 1892, Vice-President Hammer in the Chair.

ON THE LAW OF HYSTERESIS (PART II.)

AND OTHER PHENOMENA OF THE MAGNETIC CIRCUIT.

BY CHARLES PROTEUS STEINMETZ.

At the sixty-third meeting of this Institute, on January 19th, 1892, in a paper, "On the Law of Hysteresis,"[1] I have shown that the energy converted into heat during a complete cycle of magnetization can be expressed by the empirical formula

$$H = \eta B^{1.6},$$

where $\pm B$ is the maximum magnetic induction reached during the cyclic process, and η a "coefficient of hysteresis."

I have given the numerical values of this coefficient, η, for different materials, varying for

Wrought-iron, between	.002	and	.0045	
Cast-iron	.016			
Annealed steel	.008	to	.012	and up to
Hardened steel	.025	to	.082	in manganese steel
Magnetite	.020			

I have shown that this "coefficient of hysteresis," η, is apparently independent of the speed of reversals in practical limits, being the same for slow reversals as for rapid alternations up to somewhat over 200 complete periods per second. The tests published there, covered the whole range, from very low magnetization, $B = \pm 85$ lines of magnetic force per cm.^{2} up to saturations as high as $B = \pm 19{,}000$ lines of magnetic force per cm.^{2} giving fair agreement with the law of the 1.6th power.

Under conditions where eddy or Foucault currents were induced in the iron, the loss of energy followed the more general formula,

---

[1] TRANSACTIONS, vol. ix, p. 1,

$$H = \eta B^{1.6} + \varepsilon N B^2,$$

where N is the frequency, H the whole loss per cycle and cm.^{3} in ergs or absolute units, and

$H_1 = \eta B^{1.6}$ represents the loss by molecular hysteresis,
$H_2 = \varepsilon N B^2$ represents the loss by eddy-currents.

In an appendix I have shown that when the hysteretic loss H is represented as function of the M. M. F. F,

$$H = f(F),$$

we derive a curve of that shape which we would expect on the hand of the theory of molecular magnets, as formulated by Ewing.

The next question which offered itself was, to determine the conversion of energy into heat during a magnetic cycle completed between any two limits, either of opposite or of equal sign; for instance during a cyclic variation of B between $B_1 = + 10,000$ and $B_2 = - 2000$, or between $B_1 = + 18,000$ and $B_2 = + 6000$.

In the latter case Ewing, I believe on the hand of theoretical reasoning rather, contended the hysteretic loss to be very small or, in the limits of saturation, even nil.

To determine the loss of energy in a magnetic cycle between any two limits, B_1 and B_2, I have made a number of tests:

1. By the electro-dynamometer method, by employing *pulsating* currents for the excitation of the magnetizing helices; that is, currents which were derived by the superposition of an alternating and a continuous E. M. F.

2. By means of the Eickemeyer differential magnetometer, described in the former paper.

CHAPTER I. ELECTRO-DYNAMOMETER TESTS.

In the same manner as described in the former paper, a magnetic circuit of rectangular form was built up of 41 layers of sheet-iron, each layer consisting of two pieces of 20 cm. length and 2.62 cm. width, and two pieces of 7.5 cm. length and 2.62 cm. width. of the thickness $\delta = .042$ cm. (calculated from weight, specific gravity $= 7.7$).

Length of magnetic circuit,	41 cm.
Cross-section	4.512 cm.^{2}

Between the different layers, two sheets of thin paper were laid to give thorough insulation against eddy-currents. On the long sides of the rectangle forming the magnetic circuit, two magnetizing coils were wound, and connected in series, each consisting of 50 turns of three wires, No. 10 B. and S. gauge, wound simultaneously. Connecting the three wires, No. 10, in parallel gave 100 exciting turns of a resistance of .048 ω.

The instruments employed were the same as used in the former experiments, of which the constants are there given. The alternating E. M. F. was derived from the same Westinghouse 1 H. P. dynamo, varied in frequency and E. M. F., and driven in the same manner as before. In the same circuit with the Westinghouse dynamo and exciting helices, were connected in series three cells of an Eickemeyer storage battery and a rheostat.

To determine whether the superposition of the alternating E. M. F. affected the E. M. F. of the storage battery, the fixed coil of an electro-dynamometer was excited from a separate source, and the current of the storage battery sent through the movable coil, the armature of the Westinghouse dynamo and the rheostat. Then the Westinghouse dynamo was started, and it was found that the deflection of the electro-dynamometer was not changed perceptibly, thereby showing the absence of any perceptible interference between the alternating and the continuous E. M. F.'s.

The method of determination had to be changed somewhat to make it applicable to tests with pulsating current.

If the fine wire coil of the wattmeter is connected in shunt to the magnetizing helices, across the main circuit, the wattmeter measures the whole energy expended in the magnetizing helices, which consists of the energy consumed by the iron, and the energy consumed by the electric resistance of the magnetizing helices. For low and medium magnetization, the magnetizing current, and therefore the energy consumed in the electric resistance, constitutes only a small percentage of the whole wattmeter reading, and correction, therefore, can be easily made. But if a higher rate of saturation is reached, the magnetizing current becomes very large and the energy consumed by the electric resistance becomes a great or even the greater part of the whole expenditure of energy. At the same time, the temperature of the magnetizing helices rises somewhat, and consequently, the electric temperature coefficient of copper being very large, its electric resistance increases and the energy expended thereby can not be determined exactly. This impairs the exactness of the readings at higher saturation considerably.

Now, if upon the alternating E. M. F. a continuous E. M. F. is superposed, the current increases greatly, while the magnetic fluctuation and consequently the energy consumed by the iron decreases, because now the magnetic cycle is performed entirely or greatly within the limits of saturation.

For instance, while an *alternating* E. M. F. of 15.8 volts effective, at the frequency 170, sends only 1.6 amperes through the magnetic circuit described above, a *pulsating* E. M. F. of 15.8 volts effective, produced by the superposition of six volts storage battery upon an alternating E. M. F., sends not less than 14.5 amperes effective through the same magnetic circuit at the same frequency. Hence I devised another method whereby I was enabled entirely to eliminate the loss of energy caused by the electric resistance of the magnetizing helices (and of ammeter, etc.) and directly to measure the energy given off to the iron.

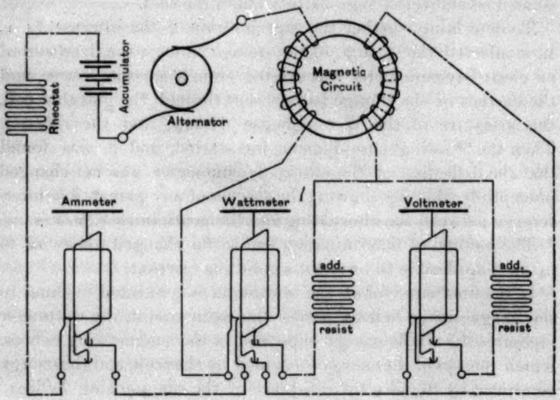

FIG. 1.—Diagram of Connections.

Of the three wires, No. 10, which were wound simultaneously on the magnetizing helices, only two were joined in parallel and connected into the main circuit, in series to ammeter, coarse wire coil of wattmeter, alternator, storage battery and rheostat. Voltmeter and fine wire coil of wattmeter, with their additional resistances, were connected into the third wire of the magnetizing helix in a separate secondary circuit, as shown in the diagram Fig. 1.

As seen, in this connection the voltmeter directly measures the E. M. F. induced by the fluctuation of the magnetism, that is, measures these fluctuations, while the wattmeter measures the time integral of the product of instantaneous values of main current into variation of magnetism,

$$\frac{1}{T}\int_{o}^{T} c\, d\, M\,,$$

that is, the energy given off to the iron. It was necessary to correct only for the small amount of energy transferred from the iron to the secondary circuit, and possible thereby to measure exactly even small magnetic fluctuations taking place at high values of saturation. The precautions taken, the method of determination and calculation of the readings, etc., were essentially the same as in the former tests, so that I need not dwell upon them.

The magnetic characteristic $B = f(F)$ derived from these tests, was checked by means of the differential magnetometer.

Tests were made at the frequencies of

170 complete periods per second,
110 " " "
67 " " "

first with alternating current, using only the alternator, then with pulsating current, having three cells of storage battery in series to the alternator, and then with pulsating currents with three cells of storage battery and rheostat in series to alternator.

The *magnetic characteristic* is given in Table I. in the usual manner, that

F = M. M. F. in ampere-turns per cm. length of magnetic circuit,
B = magnetic induction in thousands of lines of magnetic force per cm.^{2},
ρ = metallic reluctivity in thousandths, that is:

If we subtract from the magnetic induction B the magnetic field intensity $H = \frac{4\pi}{10} \sim \frac{5}{4}\, F$, and thereby derive the "metallic induction,"[1] $L = B - H$, this metallic induction is

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[1] Kennelly on Magnetic Inductance, TRANSACTIONS, vol. viii, p. 485, October, 1891.

TABLE I.

MAGNETIC CHARACTERISTIC OF SHEET-IRON IN KILOLINES.

$$\rho = 3.16\, e^{-.72\, F} + .275 + .058\, F, \text{ in mils.}$$

F.	B.	$\rho_{\text{obs.}}$	$\rho_{\text{calc.}} - \rho_{\text{obs.}}$	F.	B.	ρ.
1.	.54	1.85	+.02	16	13.32	$= .275 + .058\, F.$
1.5	1.00	1.50	−.06	18	13.67	
2	1.70	1.18	−.04	20	13.95	
2.5	2.60	.952	−.018	25	14.52	
3	3.65	.822	−.009	30	14.94	
3.5	4.74	.738	−.006	35	15.23	
4	5.86	.683	+.001	40	15.47	
4.5	6.85	.658	+.002	45	15.65	
5	7.77	.544	+.007	50	15.80	
5.5	8.55	.644	+.010	60	16.06	
6	9.27	.648	+.016	70	16.24	
6.5	9.85	.661	+.020	80	16.38	
7	10.28	.682	+.018	90	16.49	
8	10.83	.739	+.010	100	16.57	
9	11.30	.797		[120	16.71	
10	11.71	.855		150	16.86	
12	12.37	.971		200	17.09	
14	12.90	1.087		1000	18.41]	

Absolute saturation, $(B - H)_\infty = 17.24.$

$$L = \frac{F}{\rho},$$

where ρ is the "metallic reluctivity" (referred to *ampere-turns* as unit); indeed, referring to *magnetic field intensity* H as unit, we get

$$L = \frac{H}{\rho_0},$$

where

$$\rho_0 = \frac{4\pi}{10}\rho \sim \frac{5}{4}\rho.$$

Or, in the usual manner of writing, calling the "permeability" μ and the "susceptibility" $\varkappa$, we have

$$B = \mu H = (4\pi\varkappa + 1)\, H,$$

and I being the "intensity of magnetization," or "magnetic moment,"

$$I = \varkappa H, \quad \text{and}$$
$$B = 4\pi I + H,$$

so that the "metallic induction" is

$$L = 4\pi I,$$

and the "metallic reluctivity"

$$\rho_0 = \frac{1}{4\varkappa\pi} \sim \frac{2}{25\,\varkappa}.$$

In the following I shall, as in my former communication, exclusively use as unit of M. M. F., F, the "ampere-turn per cm.," since this is the unit directly derived by the tests and, at the same time, the value needed in electrical design, so that by this the factor $\frac{4\pi}{10}$ is avoided. The absolute units H and ρ_0 can easily be derived herefrom by the equations given above, $H = \frac{4\pi}{10} F$, and $\rho_0 = \frac{4\pi}{10} \rho$.

In Table I. this "metallic reluctivity" in thousandths can, over the whole range of magnetization, be expressed with fair approximation by the equation

$$\rho = 3.16\, e^{-.72\, F} + .275 + .058\, F,$$

About at $F = 7$ the first term, $3.16\, e^{-.72\, F}$, vanishes and the reluctivity assumes the simpler form

$$\rho = .275 + .058\, F,$$

given by Kennelly, in his paper already cited.

The "metallic induction" is, then,

$$L = \frac{F}{\rho},$$

and the whole induction

$$B = \frac{F}{\rho} + \frac{4\pi}{10} F;$$

where, in the range used in dynamo building, etc., the last term can usually be neglected, and instead of B using L.

This iron reaches "absolute saturation" at the "metallic induction" $L_\infty = 17.24$ kilolines.

TABLE II.

Frequency, $N = 170$ complete periods per second.

ALTERNATING MAGNETISM.

$\pm B$.	H. obs.	H. calc.	$H. - H =$ obs. calc.	%
2.74	1.17	1.11	−.06	−5
3.59	1.62	1.70	+.08	+5
3.89	1.97	1.94	−.03	−2
5.50	3.41	3.38	−.03	−1
7.52	5.61	5.57	−.04	−1
		Av.	±.05	±3
		Av. dev......	−.02	−1

TABLE III.

Frequency, $N = 110$ complete periods per second.

ALTERNATING MAGNETISM.

$\pm B.$	$H_{\text{obs.}}$	$H_{\text{calc.}}$	$H_{\text{calc.}} - H_{\text{obs.}} =$	%
1.91	.68	.62	−.06	−10
2.54	.93	.98	+.05	+ 5
2.80	1.14	1.15	+.01	+ 1
3.185	1.50	1.41	−.09	− 6
4.12	2.19	2.13	−.06	− 3
4.77	2.56	2.68	+.12	+ 4
5.82	3.75	3.69	−.06	− 2
6.48	4.25	4.39	+.14	+ 3
7.12	4.72	5.10	+.38	+ 7
7.72	5.46	5.80	+.34	+ 6
8.48	6.98	6.75	−.23	− 4
9.74	8.50	8.43	−.07	− 1
11.70	11.65	11.29	−.36	− 3
14.65	16.30	16.19	−.11	− 1
16.64	19.83	19.85	+.02	+ 0
		Av.........	±.14	±4
		Av. dev.. ...	+0	−0

TABLE IV.

Frequency, $N = 67$ complete periods per second.

ALTERNATING MAGNETISM.

$\pm B.$	$H_{\text{obs.}}$	$H_{\text{calc.}}$	$H_{\text{calc.}} - H_{\text{obs.}} =$	%
2.50	.93	.95	+.03	+2
7.22	5.40	5.22	−.18	−3
8.18	6.07	6.37	+.30	+5
		Av.	±.17	±3
		Av. dev	+.02	+1

In Tables II. III. and IV. are given the tests made with *alternating currents.*

$\pm B =$ maximum value of magnetic induction in kilolines of magnetic force per cm.^{2} The corresponding M. M. F. $\pm F$ can be taken from Table I.

$H_{\text{obs.}} =$ the observed value of the energy consumed by hysteresis during one complete cycle of magnetization, in kilo-ergs or thousands of ergs per cm.^{3} iron.

$H_{\text{calc.}} =$ the value of the energy consumed by hysteresis, calculated by means of the "coefficient of hysteresis" $\eta = .003497$.

$H_{\text{calc.}} - H_{\text{obs.}}$ gives the difference between these two values in ergs and in percentages of $H_{\text{calc.}}$.

The tests cover the range of magnetization from $B = 1910$ up to $B = 16,640$, for frequencies of 170, 110 and 67 complete periods per second.

As seen, at these speeds the "coefficient of hysteresis" is *constant*, and therefore the consumption of energy by hysteresis is still independent of the frequency.

As average of these 23 values, as coefficient of hysteresis, is derived the value

$$\eta = .003497,$$
$$\sim .0035 \qquad (1)$$

TABLE V.

Frequency, $N = 178$ complete periods per second.

PULSATING MAGNETISM.

Constant E. M. F., $V_c = 6$ volts.
Constant M. M. F., $F_c = 22.93$ ampere turns per cm.
Magnetism induced thereby, $B_c = 14.3$ kilolines per cm.^{2}

$B_{\text{obs.}} = \frac{B_1 - B_2}{2}$	$H_{\text{obs.}}$	$H_{\text{calc.}}$	$H_{\text{calc.}} - H_{\text{obs.}}$	=%	V. Volts effective.	F. Amp. turns effective.	B_1	B_2	$\frac{B_1 + B_2}{2}$
2.41	.93	.90	−.03	−3	8.4	30	+15.4	+10.6	13.0
3.12	1.35	1.36	+.01	+1	11.1	34	+15.5	+ 9.2	12.4
4.08	2.07	2.09	+.02	+1	14.6	37	+15.5	+ 7.4	11.4
7.00	5.03	4.96	−.07	−2	25.1	44	+15.6	+ 1.6	8.6
7.70	5.46	5.78	+.32	+6	26.3	47	+15.7	+ .3	8.0
		Av...	±.09	2.6					
		Av.dv	+.05	+.6					

---

1. In the appendix to the paper of January 19th, 1892, a curve of hysteresis is already given, constructed by means of a part of these tests, giving

$$\eta = .003507,$$
$$\sim .0035.$$

TABLE VI.

Frequency, $N = 115$ complete periods per second.

PULSATING MAGNETISM.

Constant E. M. F., $V_c = 6$ volts and less.

Constant M. M. F., $F_c = 22.2$ to 17.8 ampere turns per cm.

Magnetism induced thereby, $B_c = 14.15$ to 13.70 kilolines per cm.^{2}

$B =$ obs. $\frac{B_1-B_2}{2}$	H. obs.	H. calc.	$H-H$. calc obs	$=\%$	V. Volts effective.	F. Amp. turns effective.	B_1	B_2	$\frac{B_1+B_2}{2}$
1.63	.50	.48	−.02	−5	3.7	22	+15.0	+11.8	13.4
2.80	1.14	1.15	+ 01	+1	6.5	26	+15.2	+ 9.6	12.4
5.40	3.30	3.28	−.02	−1	12.1	34	+15.3	+ 4.5	9.9
5.75	3.68	3.63	−.05	−1	13.1	38	+15.3	+ 3.7	9.5
11.35	10.55	10.76	+.21	+2	25.8	42	+15.5	− 7.2	4.15
		Av ..	±.06	±2					
		Av.dv	+.03	−1					

TABLE VII.

Frequency, $N = 175$ complete periods per second.

PULSATING MAGNETISM.

Constant E. M. F., $V_c = 6$ volts.

Constant M. M. F., $F = 3.415$ ampere turns per cm.

Magnetism induced thereby, $B_c = 4.6$ kilolines per cm.^{2}

$B =$ obs. $\frac{B_1-B_2}{2}$	H. obs.	H. calc.	$H-H$. calc.obs	$=\%$	V. Volts effective.	F. Amp. turns effective.	B_1	B_2	$\frac{B_1+B_2}{2}$
1.51	.44	.43	−.01	−2	5.3	5.1	+ 6.1	+3.1	4.6
1.75	.59	.54	−.05	−9	6.0	5.3	+ 6.4	+2.9	4.6
3.31	1.54	1.50	−.04	−3	11.5	6.1	+ 8.1	+1.5	4.8
3.88	1.92	1.93	+.01	+1	13.6	7.1	+ 8.7	+ .9	4.8
5.24	3.18	3.12	−.06	−2	18.4	9.1	+10.3	− .2	5.1
		Av...	±.034	±3.4					
		Av.dv	−.03	−3					

TABLE VIII.

Frequency, $N = 111$ complete periods per second.

PULSATING MAGNETISM.

Constant E. M. F., $V_c = 6$ volts.
Constant M. M. F., $F_c = 3.49$ ampere turns per cm.
Magnetism induced thereby, $B_c = 4.7$ kilolines per cm.^{2}

$B =$ obs. $\frac{B_1 - B_2}{2}$	H. obs.	H. calc.	$H - H$. calc. obs	$= \%$	V. Volts effective.	F. Amp. turns effective.	B_1	B_2	$\frac{B_1 + B_2}{2}$
.92	.193	.193	−0	−0	2.1	3.8	+5.6	+3.8	4.7
1.86	.62	.60	−.02	−3	4.1	5.7	+6.0	+2.8	4.7
1.96	.64	.65	+.01	+2	4.3	5.7	+6.7	+2.7	4.7
2.52	1.00	.97	−.03	−3	5.5	6.7	+7.3	+2.3	4.8
		Av...	±.015	±2					
		Av.dv	−.01	−1					

In tables V., VI., VII. and VIII. are given tests made with pulsating currents at the frequencies 178 and 115, and 175 and 111.

B_1 and B_2 are the two limiting values of magnetic induction between which the cycle was performed.

Since in the alternating current tests $B =$ the amplitude of magnetic fluctuation, here as B is given half the difference between B_1 and B_2, that is, again the amplitude of magnetic variation.

$$B = \frac{B_1 - B_2}{2}.$$

The continuous E. M. F. consisted of three cells of storage battery, giving approximately $V_c = 6$ volts.

The M. M. F. of the continuous part of the current is given as F_c, and amounted to 22.93, 22.2 to 17.8, 3.415 and 3.488 ampere-turns per cm. respectively. The magnetic induction excited by this M. M F., F_c, if no alternating M. M. F. is superposed, is given by B_c, and amounted to 14.30, 14.15 $\sim$ 13.70, 4.60 and 4.70 kilolines of magnetic force per cm.^{2} respectively.

In the second set of tests the E. M. F. of the storage battery fell off somewhat.

V gives the E. M. F. of the *alternator*, which was superposed upon the $V_c = 6$ volts, in volts *effective*.

F gives the M. M. F. of the *alternating* part of the current, in *effective* ampere-turns per cm. (so that the maximum alternating M. M. F. is $= \sqrt{2} \times F$).

B_1 and B_2 give *approximate* values of the two limiting values of magnetization, and $\frac{B_1 + B_2}{2}$ their mean, calculated by means of the observed values $B = \frac{B_1 - B_2}{2}$.

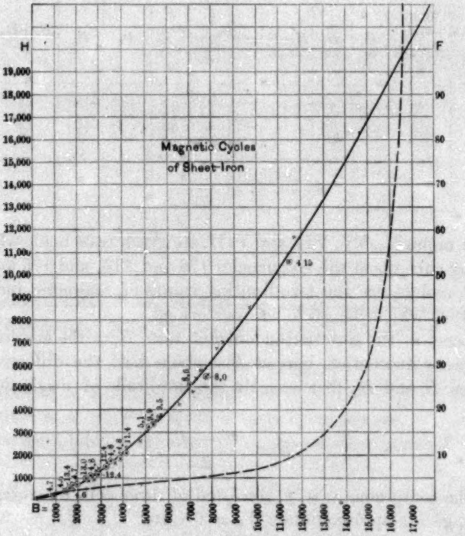

FIG. 2.—Sheet-Iron. Curve of Hysteresis.

$H_{\text{obs.}}$ = the observed value of energy consumed by hysteresis during the magnetic pulsation with the amplitude 2 B, that is, between the values B_1 and B_2, in kilo-ergs per cycle and cm.[3]

$H_{\text{calc.}}$ = the energy calculated by the formula

$$H = \eta \, B^{1.6}$$

where $B = \frac{B^1 - B^2}{2}$, and $\eta = .003497$ is the coefficient of hysteresis, found by tests with alternating currents.

$H_{\text{calc.}} - H_{\text{obs.}}$ gives again the difference in ergs and in per cents.

Fig 2 gives the curve of hysteresis, with the values observed by means of alternating currents marked by crosses +, the values observed by pulsating currents marked by circles ⊙. The average value of magnetization, $\frac{B_1 + B_2}{2}$, is written in the figure in kilolines. The dotted curve is the magnetic characteristic.

These tests prove that *the energy dissipated by hysteresis depends only upon the difference of the limiting values of magnetic induction, between which the magnetic cycle is performed, but not upon their absolute values, so that the energy dissipated by hysteresis is the same as long as the amplitude of the magnetic cycle is the same, no matter whether the cycle is performed for instance between the values of magnetization,*

$$\begin{aligned} &B_1 = + 4000 \text{ and } B_2 = - 4000, \\ \text{or } &B_1 = + 6000 \text{ and } B_2 = - 2000, \\ \text{or } &B_1 = + 8000 \text{ and } B_2 = 0, \\ \text{or } &B_1 = + 14000 \text{ and } B_2 = + 6000. \end{aligned}$$

In either case the hysteretic loss is the same, since the magnetic variation is the same, $B_1 - B_2 = 8000$.

Hence the general form of this empirical law of hysteresis is

$$H = \eta \left(\frac{B_1 - B_2}{2} \right)^{1.6},$$

where B_1 and B_2 are the values between which the magnetism varies, η a constant of the material, in our case $= .0035$.

Including the energy dissipated by eddy-currents, we derive

$$H = \eta \left(\frac{B_1 + B_2}{2} \right)^{1.6} + \varepsilon N \left(\frac{B_1 - B_2}{2} \right)^{2},$$

where N is the frequency, ε a coefficient of eddy-currents.

Herewith I conclude the first part, the results of the tests made by means of the electro-dynamometer method with alternating and with pulsating current. A large number of further tests made by the same method proved these results, but cannot be given here, since I have had no time to reduce them to absolute units.

For further tests made with alternating currents by means of the electro-dynamometer method, see Chapter IV.

CHAPTER II.—MAGNETOMETER TESTS.

A large number of tests have been made by means of the Eickemeyer differential magnetometer, of which description and illustration is found in the former paper.

To increase the sensitivity of the instrument and reach down to lower values of magnetization where the directing force of the magnetizing coil is weak enough to allow a perceptible influence of outside magnetism, the terrestrial magnetism was balanced by means of two permanent steel bar magnets of 10″ length and $\frac{3}{4}$″ cross-section.

In the tests, the direct method was used exclusively, and the tested piece balanced against standard iron of known magnetic characteristic, because the method of overbalancing the test piece by an integer number of cm.^{2} of Norway iron and then adding to the test piece as much standard iron as will restore equilibrium, is for low magnetization and test pieces of high coercitive force liable to an error introduced by the fact that the test piece is the seat of an independent M. M. F., that of the remanent magnetism, as will best be understood by comparing it with the differential galvanometer.

In determining the magnetic characteristic, before each test the magnetizing current, and therefore the magnetism, was reversed repeatedly to destroy the remanent magnetism left from former readings, and always *first readings with lower, than with higher magnetization, were taken to make sure that the remanent magnetism of the former test could be destroyed by the reversal of magnetism in the following test.*

The hysteretic curves were taken by varying the magnetizing current cyclic and taking readings at every step. Usually two or three complete cycles were taken, plotted on cross-section paper, and the values of the magnetization from 5 to 5 taken from the plotted curve, or from 10 to 10 ampere turns per cm., and these values added together, which gave the value of H. Before the readings a larger number of cycles were performed to make sure that during the readings the cyclic process had become stationary already.

In some cases a differential method was used, by balancing the test piece against another piece of similar magnetic characteristic, which had been tested before, and was in this way used as an auxiliary standard.

TABLE IX.

MAGNETIC CHARACTERISTIC OF THIN TIN-PLATE.

30 pieces = 2 05 cm.^{2}

C.	$s+a$	F.	S.	A.	$M. = Ss + Aa$	$L. = \frac{M.}{2.05.}$	$\rho_{obs.} = \frac{F.}{L.}$	$\rho_{calc.} = .192 + .05464\ F.$	$\rho_{calc} - \rho_{obs.}$	= %	H.	B.
.45	$1+16$	8	13.30	540	21.94	10.70	.748	(.629)	.. .. ..	.... ..	.01	10.71
.55	$2-2\frac{1}{4}$	10.5	14.20	595	27.06	13.19	.798	(.766)	.. ....	....	.01	13.20
.80	$2-\frac{7}{16}$	14	15.10	645	29.92	14.59	.960	.957	−.003	−.3	.02	14.61
1.15	$2-\frac{1}{16}$	20	16.00	695	31.96	15.58	1.284	1.285	+.001	+.1	.03	15.61
1.40	$2+\frac{1}{8}$	26	16.47	730	33.02	16.10	1.616	1.613	−.003	−.2	.03	16.13
1.70	$2+\frac{7}{16}$	34	16.90	758	34.16	16.65	2.04	2.05	+.01	+.5	.04	16.69
2.20	$2+\frac{1}{2}$	47	17.30	781	34.98	17.05	2.76	2.76	0	0	.06	17.11
2.90	$2+\frac{1}{2}$	62	17.57	802	35.54	17.33	3.58	3.58	0	0	.08	17.41
4.4	$2+\frac{5}{8}$	85	17.78	818	36.08	17.59	4.84	4.84	0	0	.11	17.70
5.6	$2+\frac{11}{16}$	97	17.83	821	36.22	17 66	5.49	5.49	0	0	.12	17.78
7.5	$2+\frac{3}{4}$	110	17.89	825	36.40	17.74	6.20	6.20	0	0	.14	17.88
10.5	$2+\frac{3}{4}$	124	17.94	829	36.50	17.79	6.97	6.97	0	0	.16	17.95
18	$2+\frac{3}{4}$	143	18.02	832	36.66	17.87	8.00	8.01	+.01	+.1	.18	18 05
								Av. =	±.0025	= ±.1		

$$F \geqq 14. \qquad \rho = .192 + .05464\ F.$$

As an example, I give in Table IX. a set of tests made for determining the magnetic characteristic of a sample of thin tin-plate,of which 30 pieces were used, of 2.55 cm. width and .0268 cm.thickness, giving 2.05 cm.^{2} cross-section.

C = current in the magnetizing coil of the magnetometer.

$s + a$ = number of cm.^{2} Norway iron (s) and of pieces of soft sheet-iron (a), of $\frac{1}{20}$ cm.^{2} cross-section, necessary to balance the test piece.

F = M. M. F. in ampere turns per cm., corresponding to current C and reluctance $s + a$, taken from the characteristic curves of the instrument.

S and A are the number of lines of magnetic force which a cm.^{2} Norway iron (s) or $\frac{1}{20}$ cm.^{2} sheet-iron (a) carry respectively at the M. M. F., F.

$M = s\,S + a\,A$ is consequently the number of lines of magnetic force carried by $s + a$ and therefore by the test piece. Hence

$L = \frac{M}{2.05} = \frac{s\,S + a\,A}{2.05}$ is the (metallic) magnetic induction in the test piece.

$\rho_{obs.} = \frac{F}{L}$ is the metallic reluctivity of the test piece which, for

$$F \geqq 14,$$

can be expressed by the equation, derived from these tests,

$$\rho = .192 + .05464\ F.$$

$\rho_{\text{calc.}}$ is the value of metallic reluctivity calculated from this equation, and

$\rho_{\text{calc.}} - \rho_{\text{obs.}}$ the difference in absolute values and in percentage of $\rho_{\text{calc.}}$.

$H = \frac{4\pi}{10} F$ is the field intensity, corresponding to M. M. F., F, and thus

$$B = L + H,$$

the whole magnetic induction in the test piece.

It must be understood that the differential magnetometer measures *not* the whole induction B, but the *metallic induction*

$$L = B - H = 4\pi\kappa H.$$

In all the following tests, NOT *the whole induction B, but the metallic induction L is given. To determine, therefore, the whole induction B, the field intensity $H = \frac{4\pi F}{10}$ has to be added.*

For the value of hysteresis, the addition of H makes no difference, since space has no hysteresis.

Where the dimensions of the test piece are not given, they are cylindrical pieces of 4 cm.^{2} cross-section and 20 cm. length, fitting into the pole-blocks of the magnetometer.

TESTS.

I. Cast-Iron.

1. *Ordinary Cast-Iron.*

Table X. gives the magnetic characteristic in the first column.

TABLE X.

MAGNETIC CHARACTERISTICS OF GRAY CAST-IRON.

	No. 1.		No. 4.		No 7. ½% Al.		No. 8. ½% Al.	
F.	*ρ.*	*L.*	*ρ.*	*L.*	*ρ.*	*L.*	*ρ.*	*L.*
7.5	6.20	1.21	3.98	1.92	6.80	1.10	8.20	.92
10	5.00	2.00	3.70	2.70	5.45	1.84	6.55	1.53
12.5	4.20	2.98	3.58	3.49	4.50	2.78	5.40	2.31
15	3.94	3.81	3.59	4.17	4.13	3.63	4.80	3.12
17.5	4.05	4.33	3.72	4.73	4.16	4.21	4.70	3.73
20	$\rho = 2.40 + .09400 F.$	4.68	$\rho = 2.05 + .09725 F.$	5.00	$\rho = 2.37 + .0976 F.$	4.63	$\rho = 2.92 + .0948 F.$	4.15
30		5.74		6.04		5.67		5.20
40		6.50		6.72		6.37		5.96
50		7.05		7.23		6.90		6.52
60		7.46		7.60		7.30		6.97
80		8.06		8.13		7.87		7.61
100		8.47		8.50		8.25		8.07
150		9.10		9.00		8.81		8.74
[200		9.42		9.31		9.14		9.14
300		9.81		9.60		9.48		9.57
400		10.00		9.77		9.66		9.80
500		10.12		9.89		9.78		9.93]
Absolute saturation		10.66	....	10.28	....	10.25	....	10.55

$F =$ M. M. F. in ampere turns per cm.
$L =$ metallic induction in thousands of lines of force per cm.^{2}
$\rho =$ metallic reluctivity $\dfrac{F}{L}$ in thousandths (10^{-3}).

The values inclosed in brackets are extrapolated by means of the law

$$\rho = a + \sigma F \qquad \text{[Kennelly, paper before cited].}$$

Tables XI. and XII. give 11 magnetic cycles of this cast-iron and Table XIII. the results of these cycles.

TABLE XI.

HYSTERESIS OF ORDINARY GRAY CAST-IRON, No. 1.

F.	(1)		(2)		(3)		(4)		(5)	
	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r
+44			± 6.68		+ 6.70		+ 6.70		+ 6.70	
40			6.58	6.44	6.60	6.52	6.60	6.54	6.60	6.56
35			6.42	6.10	6.46	6.22	6.46	6.25	6.46	6.32
30			6.20	5.70	6.28	5.91	6.28	5.93	6.28	6.03
25			5.93	5.10	6.01	5.45	6.01	5.51	6.01	5.66
20			5.60	4.35	5.70	5.00	5.70	5.26	5.70	5.50
15	± 3.40		5.17	3.00	5.30	4.35	5.30	5.13	+ 5.33	
10	2.92	1.60	4.58	.70	4.80	3.40	+ 4.66		[$F_2 = +16.$]	
+ 5	2.35	− .55	3.80	− 1.40	4.20	1.00	[$F_2 = +11.$]			
0	± 1.60		± 2.80		3.10	0				
− 5					1.60	− .25				
− 9					− .32					
$H =$	5.82		17.08		6.13		.86		.48	
$L =$	3.40		6.68		3.51		1.02		.685	
$\eta =$	.01302		.01297		.01303		.01320		.01393	

TABLE XII.

HYSTERESIS OF ORDINARY GRAY CAST-IRON, No. 1.

F.	(6)		(7)		(8)		(9)		(10)		(11)	
	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r
140					± 9.01		+ 9.06		+ 9.06		+ 9.06	
130					8.92	8.88	8.97	8.94	8.97	8.96	8.97	8.96
120					8.81	8.72	8.86	8.79	8.86	8.84	8.86	8.84
110			± 8.71		8.70	8.56	8.75	8.65	8.75	8.72	8.75	8.72
100			8.54	8.50	8.59	8.39	8.64	8.50	8.64	8.57	8.64	8.60
90			8.37	8.28	8.50	8.24	8.55	8.35	8.55	8.44	8.56	8.49
80	[$F = \pm 74.$]		8.20	8.05	8.34	8.04	8.39	8.11	8.39	8.23	8.42	8.30
70	± 7.92		8.05	7.76	8.16	7.74	8.21	7.83	8.21	8.01	8.26	8.11
60	7.62	7.44	7.80	7.40	7.96	7.36	8.01	7.51	8.02	7.78	8.08	7.92
50	7.38	6.93	7.55	6.90	7.68	6.80	7.73	7.08	7.74	7.48	7.80	7.70
40	7.06	6.37	7.20	6.35	7.34	6.36	7.39	6.61	7.41	7.16	+ 7.26	
30	6.60	5.68	6.75	5.65	6.86	5.70	6.96	6.01	7.00	6.84	[$F_2 = +40.$]	
20	5.95	4.32	6.10	4.25	6.16	4.51	6.31	5.21	+ 6.09			
10	4.90	.60	5.00	.40	4.95	1.20	5.17	3.50	[$F_2 = +17.$]			
0	± 3.03		± 3.15		± 3.30		3.40	.80				
− 9							0					
$H =$	22.46		26.34		27.54		9.19		1.51		.72	
$L =$	7.92		8.71		9.01		4.53		1.485		.90	
$\eta =$	.01298		.01308		.01295		.01299		.01288		.01350	

TABLE XIII.

HYSTERESIS OF ORDINARY GRAY CAST-IRON, No. 1—RESULTS.

No.		F_1	F_2	$F = \frac{F_1 - F_2}{2}$	L_1	L_2	$L = \frac{L_1 - L_2}{2}$	H	η	$10^5 \times \triangle\eta$	= %
(1)	*a*	+ 15	− 15	15	+3.40	−3.40	3.40	5.82	.01302	− 2	− .2
(2)	*a*	+ 44	− 44	44	+6.68	−6.68	6.68	17.08	.01297	+ 3	+ .2
(3)	*p*	+ 44	− 9	26.5	+6.70	− .32	3.51	6.13	.01303	− 3	− .2+
(4)	*p*	+ 44	+ 11	16.5	+6.70	+4.66	1.02	.86	.01320	−20	−1.5+
(5)	*p*	+ 44	+ 16	14	+6.70	+5.33	.685	.48	.01393	−93	−7.1+
(6)	*a*	+ 74	− 74	74	+7.92	−7.92	7.92	22.46	.01298	+ 2	+ .1
(7)	*a*	+110	−110	110	+8.71	−8.71	8.71	26.34	.01308	− 8	− .6
(8)	*a*	+140	−140	140	+9.01	−9.01	9.01	27.54	.01295	+ 5	+ .4
(9)	*p*	+140	− 9	74.5	+9.06	0	4.53	9.19	.01299	+ 1	+ .1+
(10)	*p*	+140	+ 17	61.5	+9.06	+6.09	1.485	1.53	.01288	+12	+ .9+
(11)	*p*	+140	+ 40	50	+9.06	+7.26	.90	.72	.01350	−50	−3.8+
								Av..	.01300		

Here are

F_1 and F_2, the maximum and the minimum value of M. M. F. in ampere turns per cm.

L_1 and L_2, the maximum and the minimum value of magnetic induction in kilolines of magnetic force per cm.^{2}

$F = \frac{F_1 - F_2}{2}$, the amplitude of variation of M. M. F.

$L = \frac{L_1 - L_2}{2}$, the amplitude of variation of magnetic induction.

H, the observed value of hysteretic dissipation of energy in kilo-ergs per cycle and cm.^{3}

η, the coefficient of hysteresis calculated therefrom.

$\triangle$, the difference between this observed value of η and the average of η taken from the five largest cycles (since in small cycles the exactness is necessarily considerably smaller, the result being based upon a lesser number of readings, I deemed it advisable to use only the largest cycles for the calculation of the mean value of η).

The conclusion derived from these tests is the same as that derived from the electro-dynamometer tests, namely, that the loss of energy by hysteresis can be expressed by the equation

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6}.$$

Hence the magnetic properties of this cast-iron can be expressed by means of the equations

$$\rho = \alpha + \sigma F,$$
$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6},$$

by three constants,

α, the "*coefficient of magnetic hardness,*"
σ, the "*coefficient of magnetic saturation,*"
η, the "*coefficient of magnetic hysteresis.*"

Only for values of $F \leqq 20$ the value of ρ, if determined by *reversals* of magnetism, is larger and may necessitate the introduction of a term, $c\,e^{-\delta F}$, or of similar shape.

The term α I call the "coefficient of magnetic hardness," since the value of α determines what is called "magnetically hard." I shall still show in the following that α is smallest in soft Norway iron, increases by hardening and reaches very large values in glass-hard steel.

The term σ I call the "coefficient of magnetic saturation," because $L_\infty = \frac{1}{\sigma}$ is the value of absolute saturation of the metallic induction, that is, the value which the metallic induction reaches for infinitely large M. M. F's that is, for values larger than $F = 1000$ to 20,000 (according to the value of magnetic hardness α).

2. *Cast-Iron with $\frac{1}{8}$ %, viz., $\frac{1}{2}$ % Aluminium*[1]

(Here the tests were made by comparing the two test pieces with the cast-iron given in 1.)

Table X. gives the magnetic characteristic in the third column; Table XIV. gives two magnetic cycles of the sample containing $\frac{1}{8}$ per cent. aluminium.

Table X. gives the magnetic characteristic in the fourth column; Table XV. gives two magnetic cycles of the sample containing $\frac{1}{2}$ per cent. aluminium.

---

[1] Derived from Cornell University; a sample containing no aluminium could not be tested, because it was too hard to be turned off to standard size.

TABLE XIV.

HYSTERESIS OF CAST-IRON CONTAINING $\frac{1}{8}$ % ALUMINIUM.

	(1)			(2)	
F	L_d	L_r	F	L_d	L_r
44	± 6.49		110	± 8.48	
40	6.40	6.26	100	8.32	8.27
35	6.25	5.93	90	8.16	8.06
30	6.03	5.54	80	8.00	7.84
25	5.78	4.95	70	7.86	7.56
20	5.46	4.18	60	7.63	7.22
15	5.04	2.80	50	7.40	6.76
10	4.48	.55	40	7.08	6.23
5	3.68	— 1.50	30	6.65	5.51
0	± 2.67		20	6.01	4.04
			10	4.91	.18
			0	± 2.90	
$H =$	17.07			26.50	
$L =$	6.49			8.48	
$\eta =$	.01358			.01373	

Av. $\eta = .01365$.

TABLE XV.

HYSTERESIS OF CAST-IRON CONTAINING $\frac{1}{2}$ % ALUMINIUM.

	(1)			(2)	
F	L_d	L_r	F	L_d	L_r
44	± 6.15		110	± 8.33	
40	6.05	5.90	100	8.16	8.11
35	5.89	5.55	90	7.98	7.88
30	5.67	5.14	80	7.80	7.64
25	5.41	4.53	70	7.65	7.34
20	5.09	3.77	60	7.39	6.97
15	4.68	2.43	50	7.13	6.44
10	4.14	.28	40	6.78	5.84
5	3.45	— 1.52	30	6.32	5.07
0	± 2.60		20	5.67	3.59
			10	4.61	— .03
			0	± 3.00	
$H =$	16.89			27.28	
$L =$	6.15			8.33	
$\eta =$	.01463			.01455	

Av. $\eta = .01459$.

The denotations are the same as in the former set of tests (1).

3. *Different Samples of Cast-Iron.*

In like manner, five other samples of common cast-iron, obtained from different foundries, were tested. They are marked with 2, 3, 4, 5, 6, while the two samples of aluminium cast-iron were marked with 7 and 8. Only one cycle of each of these five samples was taken and the magnetic characteristic determined.

Of sample No. 4 the magnetic characteristic is given in the second column of Table X. Of the four other samples, Nos. 2, 3, 5 and 6, the magnetic reluctivity ρ is given in Table XVI.

TABLE XVI.

MAGNETIC RELUCTIVITY OF GRAY CAST-IRON.

	No. 2.	No. 3.	No. 5.	No. 6.
F	ρ	ρ	ρ	ρ
7.5			5.50	4.95
10.	5.15	5.40	4.60	4.10
12.5	4.35	4.65	4.10	3.68
15.	4.08	4.32	4.00	3.57
17.5	4.12	4.44	4.04	3.76
20.				
	$\rho = 2.76 + .0954\ F$	$\rho = 2.43 + .0943\ F$	$\rho = 2.34 + .0950\ F$	$\rho = 2.07 + .0972\ F$

The results of the cyclic tests of all the eight cast-iron samples are combined in Table XVII.

TABLE XVII.

MAGNETIC HYSTERESIS OF CAST-IRON—RESULTS.

	$\pm F$	$\pm L$	H	η
No. 1........ ...	Graded Cycles			.01300
No. 2	58	7.35	20.22	.01317
No. 3............	58	7.00	22.39	.01577
No. 4....	110	8.63	22.47	.01132
No. 5	110	8.60	25.01	.01267
No. 6............	110	8.62	24.17	.01222
No. 7, ⅛ per c. Al.	44	6.49	17.07	.01365
" "	110	8.48	26.50	
No. 8, ½ per c. Al.	44	6.15	16.89	.01459
" "	110	8.33	27.28	

These tests prove conclusively that beyond a certain minimum value of M. M. F., $F = 18$ to 20 ampere turns per cm., the metallic magnetic reluctivity ρ (inverse value of $\frac{16\,\pi^2}{10}x$, where x is the magnetic susceptibility) rigidly follows a straight line, $\rho = \alpha + \sigma F$, showing that the metallic induction, $L = B - H$, approaches, for infinitely high M. M. F's., as limit of absolute magnetic saturation,

$$L_\infty = \frac{1}{\sigma} .$$

Hence, beyond a minimum value of M. M. F., all the magnetic properties of cast-iron can be expressed by three constants, the

Coefficient of magnetic hardness, α;
Coefficient of magnetic saturation, σ;
Coefficient of magnetic hysteresis, η.

These three coefficients are given for the eight tested samples of cast-iron in Table XVIII., together with the absolute saturation $L_\infty = \frac{1}{\sigma}$ and the minimum value F, where ρ coincides with the straight line.

TABLE XVIII.

MAGNETIC CONSTANTS OF CAST-IRON.

	$F \geqq$	Coefficient of Magnetic Hardness α	Coefficient of Magnetic Saturation σ	Coefficient of Magnetic Hysteresis η	Absolute Saturation $L_\infty = \frac{1}{\sigma}$
No. 1............	20	2.40	.0940	.01300	10.66
No. 2............	20	2.43	.0943	.01317	10.60
No. 3.....	20	2.76	.0954	.01577	10.48
No. 4......	18	2.05	.09725	.01132	10.28
No. 5............	18	2.34	.0950	.01267	10.55
No. 6	18	2.07	.0972	.01220	10.29
No. 7, ⅛ per ct. Al.	20	2.37	.0976	.01365	10.25
No. 8, ½ per ct. Al.	20	2.92	.0948	.01459	10.55
Average.......		2.4	.096	.013	10.50

Furthermore, these tests prove that for cast-iron the dissipation of energy during a complete magnetic cycle between the limits L_1 and L_2 is expressed by the equation

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6}.$$

The cycles 1, 2. 6 and 7 of Table XI., made between opposite and equal limits of M. M. F. on cast-iron No. 1., are shown in Fig. 3.

Fig. 4 gives the cycles 2, 3, 4 and 5 of Table XI., referring also to cast-iron No. 1.

The results of all the 11 magnetic cycles of cast-iron No. 1 are shown in Fig. 5. The drawn line is the curve of hysteresis, $H = .013 \left(\frac{L_1 - L_2}{2}\right)^{1.6}$.

The observed values are marked by crosses $+$, when taken between opposite and equal limits, $L_1 = - L_2$; by circles ⊙, when taken between unequal limits of M. M. F. In the latter case the average magnetization, $\frac{L_1 + L_2}{2}$, is written in Fig. 5. The dotted line represents the magnetic characteristic.

Further cast-iron characteristics are shown in Fig. 17.

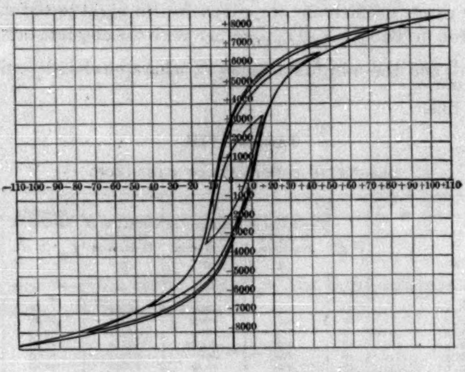

Fig. 3.—Cast-Iron. Hysteretic Cycles.

II. Tool Steel of Different Degrees of Hardness.

To determine the influence of hardening upon the magnetic constants, three pieces were cut from the same rod of tool steel, turned off cylindrical to 15 cm. length and 1 cm.^{2} cross-section, and then the one piece was annealed, the second piece was heated and hardened in oil, the third piece hardened in cold water and thereby made glass-hard. To reach higher M. M. F. than possible with test pieces of 4 cm.^{2} cross-section and the instrument at my disposition, the pole-faces of the magnetometer were brought closer together, to 6.35 cm. distance, and only 1 cm.^{2} of test piece used, whereby M. M. F.'s. up to $F = 350$ ampere turns, that is, field intensities up to $H > 400$, were available.

The test pieces were laid in holes in the pole-faces of the magnetometer, of 1 cm.^{2} cross-section, and after a *preliminary* determination of their magnetic characteristic, a number of magnetic cycles were completed with each of them between different limiting values of *F*.

Then all the three samples were found permanently and strongly magnetized. Hence, I demagnetized them by means of a powerful alternating current in the following manner:—A wire spool was slipped over each piece, and solid Norway iron blocks laid against its ends to concentrate the alternating magnetism through the whole length of the piece and to afford low transient reluctance from piece to air. Then, with a frequency of about 170 complete periods per second, an alternating current was sent through the wire spool, representing about 5000 to 6000 ampere-turns. The test piece got rather hot after some minutes' application of the alternating current, but, nevertheless, in the glass-hard piece the permanent magnetism was not fully destroyed even yet by this alternating magnetic strain, but the cycles taken with it were afterwards found unsymmetrical.[1]

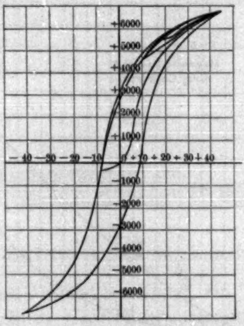

FIG. 4.—Cast-Iron. Hysteretic Cycles.

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[1] This sample of glass-hard steel was the only one which I was not able to demagnetize by a rapidly alternating M. M. F. Otherwise an alternating M. M. F. of 3000 to 4000 ampere-turns I found always able to destroy remanent and permanent magnetism within a few minutes so completely that not the least trace could be discovered.

Nevertheless, the magnetic constants of all the three pieces were found considerably changed in the way a partial annealing would do it.

Then the magnetic characteristic of each piece was determined by the method of reversals, that is, by reversing the magnetism repeatedly before each reading, since this seems to be the only method which gives *constant* and therefore *reliable* results, while the determination of the curve of rising magnetism becomes, especially for small M. M. F.'s., unreliable because of not giving always the same value for the same M. M. F.; and then again a number of cycles completed with either of the pieces.

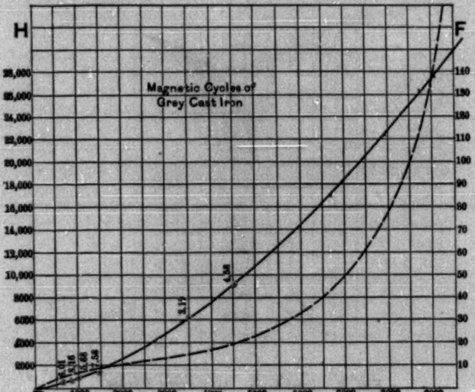


FIG. 5.—Cast-Iron. Curve of Hysteresis.

The three pieces are marked with *H*—glass-hard,
O—oil-hardened,
S—annealed,

and the values derived before the application of the alternating current marked with an *h*: *Hh*, *Oh*, *Sh*.

Unfortunately, before the application of the alternating current the magnetic characteristics had been determined only preliminarily, so that the values given therefor can be considered only as approximations, but sufficiently near to allow perceiving the influence fo the application of the alternating current.

Table XIX. gives the magnetic characteristics of the three samples in their two states.

TABLE XIX.

MAGNETIC CHARACTERISTICS OF TOOL-STEEL.

	Hh		*H*		*Oh*		*O*		*Sh*		*S*	
F	ρ	*L*	ρ	*L*	ρ	*L*	ρ	*L*	ρ	*L*	ρ	*L*
20							9.0	2.22			3.0	6.67
30			27.0	1.11	5.8	5.18	4.9	6.12	3.8	7.90	3.2	9.37
40			23.0	1.74	5.4	7.40	4.7	8.50	4.0	10.00	3.6	11.10
50	23.0	2.17	20.0	2.50	5.6	8.94	5.0	10.00	4.45	11.24	$\rho = 1.22 + .0575 F$	12.20
60	21.0	2.86	18.0	3.33	6.05	9.92	5.3	11.34	5.0	12.00		12.87
70	19.5	3.58	17.3	4.04	6.6	10.60	$\rho = 1.54 + .060 F$	12.20	$\rho = 1.33 + .060 F$	12.68		13.34
80	18.5	4.25	17.2	4.65	$\rho = 1.9 + .066 F$	11.10		12.60		13.05		13.75
90	19.0	4.74	17.5	5.15		11.54		13.00		13.37		14.08
100	$\rho = 8.0 + .121 F$	5.00	$\rho = 7.8 + .105 F$	5.47		11.76		13.25		13.67		14.37
150		5.75		6.40		12.70		14.25		14.50		15.16
200		6.22		6.95		13.25		14.78		15.04		15.75
250		6.54		7.35		13.60		15.13		15.33		16.05
300		6.78		7.64		13.80		15.40		15.55		16.25
[400		7.08		8.02		14.15		15.68		15.80		16.52]
[500		7.30		8.30		14.34		15.90		16.00		16.72]
Absolute saturation		8.28		9.53		15.16		16.70		16.70		17.40

$F =$ M. M. F. in ampere-turns per cm.

$L =$ metallic induction in thousands of lines of magnetic force per cm.^{2}

$\rho =$ metallic reluctivity $= \dfrac{F}{L}$ in thousandths.

The samples are denoted by *Hh*, *H*, *Oh*, *O*, *Sh*, *S*.

The tables XX. to XXVII. give magnetic cycles performed with the pieces, and Table XXVIII. the results of these cycles.

TABLE XX.

HYSTERESIS OF TOOL-STEEL *Hh*.

F	(1) L_d	(1) L_r	(2) L_d	(2) L_r	(3) L_d	(3) L_r	(4) L_d	(4) L_r
+275	± 5.93		+ 5.94		+ 5.95		+ 5.97	
260	5.92	5.91	5.93	5.90	5.94	5.93	5.96	5.95
240	5.91	5.83	5.92	5.85	5.93	5.86	5.95	5.93
220	5.90	5.78	5.91	5.80	5.92	5.80	5.94	5.91
200	5.89	5.70	5.90	5.72	5.91	5.74	5.93	5.88
180	5.88	5.60	5.89	5.64	5.90	5.68	5.92	5.85
160	5.87	5.45	5.88	5.52	5.88	5.58	5.90	5.80
140	5.82	5.28	5.84	5.40	5.84	5.46	5.87	5.74
120	5.73	5.10	5.79	5.21	5.78	5.29	5.82	5.68
100	5.61	4.70	5.70	4.98	5.68	5.09	5.75	5.60
80	5.43	4.10	5.57	4.47	5.55	4.75	5.60	5.43
60	5.20	2.90	5.28	3.53	5.33	4.22	5.38	5.18
40	4.80	.40	4.90	1.75	4.97	3.35	5.00	4.82
+ 20	4.10	−1.90	4.36	− .70	4.47	2.15	+ 4.66	
0	± 3.30		3.60	−1.86	3.80	1.00	[$F_2 = + 30$]	
− 20			2.37	−2.63	2.50	.45		
− 40			.30	−3.12	.40	.07		
− 60			−1.75	−3.51	0			
− 83			− 3.76		[$F_2 = - 45$]			
H =	82.04		59.04		26.52		2.42	
L =	5.93		4.85		2.975		.655	
η =	.07533		.07480		.07342		.07546	

TABLE XXI.

HYSTERESIS OF TOOL-STEEL *Hh*.

F	(5) L_d	(5) L_r	(6) L_d	(6) L_r	(7) L_d	(7) L_r
+124	± 5.12		+ 5.13		+ 5.17	
110	5.09	4.90	5.12	4.95	5.13	5.04
100	5.06	4.64	5.10	4.77	5.11	4.92
90	5.00	4.35	5.06	4.56	5.07	4.80
80	4.90	4.00	5.00	4.30	5.01	4.68
70	4.79	3.40	4.90	4.05	4.91	4.56
60	4.65	2.60	4.75	3.75	4.76	4.44
50	4.50	1.60	4.60	3.33	4.61	4.32
40	4.30	.40	4.43	2.90	4.44	4.16
30	4.05	−1.00	4.22	2.33	4.23	4.03
20	3.80	−1.90	4.00	1.73	+ 3.88	
+10	3.45	−2.55	3.75	1.20	[$F_2 = + 15.$]	
0	± 3.10		3.40	.75		
−10			3.00	.45		
−20			2.30	.25		
−30			1.20	.10		
−41			0			
H =	64.50		21.46		2.36	
L =	5.12		2.565		.645	
η =	.07493		.07533		.07560	

TABLE XXII.

HYSTERESIS OF TOOL-STEEL *H*.

	(1)		(2)		(3)		(4)		(5)		(6)	
F	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r
+120	+6.25		+6.25		+6.25				+6.25			
110	6.15	6.00	6.16	6.00	6.23	6.13			6.23	6.13		
100	6.03	5.65	6.04	5.73	6.18	6.00			6.18	6.00		
90	5.90	5.15	5.90	5.42	6.11	5.86			6.11	5.87		
80	5.72	4.50	5.75	5.08	6.02	5.71			6.02	5.72		
70	5.53	3.50	5.58	4.70	5.89	5.57			5.89	5.59		
60	5.32	2.50	5.42	4.30	5.73	5.42			5.61	5.45		
50	5.06	1.24	5.22	3.80	5.56	5.27			+5.25			
40	4.80	0	5.00	3.20	5.30	5.11			[$F_2 = +47.$]			
30	4.50	−1.15	4.75	2.50	+4.95							
20	4.15	−1.95	4.45	1.80	[$F_2 = +28.$]							
+10	3.72	−2.60	4.12	1.25								
0	3.30	−3.05	3.70	.77								
−10	2.62	−3.50	3.20	.42								
−20	1.84	−3.85	2.55	.20			[$F_1 = -28$]					
−30	.75	−4.16	1.55	.03			−4.57					
−40	−.55	−4.43	.50	−.08			5.01	4.68			[$F_1 = -47.$]	
−50	−1.75	−4.68	−.12				5.22	4.84			−4.93	
−60	−2.90	−4.90	[$F_2 = -47$]				5.37	5.00			5.32	5.08
−70	−3.84	−5.10					5.50	5.19			5.52	5.23
−80	−4.60	−5.29					5.60	5.36			5.63	5.38
−90	−5.08	−5.45					5.70	5.52			5.73	5.53
−100	−5.40	−5.60					5.80	5.67			5.82	5.67
−110	−5.64	−5.72					5.86	5.80			5.87	5.82
−120	−5.80						−5.90				−5.90	
$H =$	68.52		24.68		1.96		2.03		1.29		1.21	
$L =$	6.025		3.185		.65		.665		.50		.485	
$\eta =$	.06136		.06124		.06188		.06178		.06197		.06103	

TABLE XXIII.

HYSTERESIS OF TOOL-STEEL *Oh*.

	(1)		(2)		(3)	
F	L_d	L_r	L_d	L_r	L_d	L_r
+260	± 13.25		+ 13.25		+ 13.25	
240	13.22	13.19	13.22	13.19	13.22	13.21
220	13.19	13.10	13.19	13.10	13.19	13.17
200	13.15	13.00	13.15	13.01	13.15	13.12
180	13.10	12.88	13.10	12.90	13.10	13.06
160	13.05	12.77	13.05	12.80	13.05	12.99
140	12.99	12.66	13.00	12.70	13.00	12.92
120	12.85	12.40	12.87	12.46	12.88	12.76
100	12.66	12.03	12.68	12.12	12.69	12.50
80	12.42	11.50	12.45	11.62	12.47	12.15
60	11.95	10.30	12.00	10.50	12.02	11.65
40	11.00	7.00	11.20	8.60	11.22	11.04
+20	9.40	− 1.70	9.80	4.60	+ 10.60	
0	± 6.70		7.20	− .30	[$F_2 = +30.$]	
−20			1.80	− 1.90		
−30			+ 2 24			
$H =$	106.20		44.78		2.75	
$L =$	13.25		7.745		1.325	
$\eta =$	.02695		.02683		.02778	

TABLE XXIV.

HYSTERESIS OF TOOL-STEEL *Oh.*

	(4)		(5)		(6)	
F	L_d	L_r	L_d	L_r	L_d	L_r
+80	± 11.30		+11.30		+11.30	
70	11.10	10.70	11.10	10.75	11.10	10.82
60	10.85	9.85	10.85	10.10	10.90	10.50
50	10.55	8.75	10.55	9.30	10.68	10.22
40	10.10	6.60	10.10	8.20	10.26	9.95
30	9.50	2.70	9.55	6.70	9.48	
20	8.60	− 1.20	8.70	4.30	[$F_2 = +27$]	
+10	7.50	− 4.30	7.60	2.00		
0	± 6.00		6.20	.60		
−10			4.30	− .20		
−20			1.60	− .60		
−26			−.70			
$H =$	82.20		28.96		1.48	
$L =$	11.30		6.00		.91	
$\eta =$	.02692		.02611		.02727	

TABLE XXV.

HYSTERESIS OF TOOL-STEEL *O.*

	(1)		(2)		(3)	
F	L_d	L_r	L_d	L_r	L_d	L_r
112	±13.65		+13.65		+13.64	
100	13.54	13.32	13.54	13.35	13.54	13.40
90	13.40	12.88	13.40	13.05	13.40	13.16
80	13.22	12.32	13.22	12.74	13.22	12.93
70	13.00	11.72	13.00	12.42	13.00	12.71
60	12.70	10.90	12.70	12.08	12.70	12.46
50	12.30	9.50	12.30	11.70	12.30	12.16
40	11.75	7.00	11.75	11.28	+11.95	
30	11.00	2.50	11.09	10.80	[$F_2 = +43$]	
20	10.10	− 2.90	+10.48			
10	9.00	− 5.55	[$F_2 = +24$]			
0	±7.50					
$H =$	111.64		3.52		1.32	
$L =$	13.65		1.585		.85	
$\eta =$	.02700		.02669		.02713	

TABLE XXVI.

HYSTERESIS OF TOOL-STEEL *Sh*.

	(1)		(2)		(3)	
F	L_d	L_r	L_d	L_r	L_d	L_r
+240	±16.60		+16.60		+16.60	
220	16.58	16.52	16.58	16.53	16.58	16.57
200	16.52	16.40	16.52	16.42	16.52	16.50
180	16.45	16.27	16.45	16.30	16.45	16.41
160	16.38	16.10	16.38	16.13	16.38	16.32
140	16.28	15.90	16.28	15.95	16.28	16.20
120	16.17	15.60	16.17	15.68	16.17	16.06
100	15.95	15.20	15.95	15.30	15.95	15.78
80	15.66	14.70	13.66	14.90	15.68	15.35
60	15.20	13.30	15.20	13.60	15.25	14.80
40	14.20	9.60	14.25	10.80	14.35	14.00
20	12.00	2.10	12.40	4.50	+13.00	
0	±7.20		8.20	−3.20	[$F_2 = +26$]	
−20			1.50	−7.50		
−26			−8.00			
$H =$	108.00		66.00		3.16	
$L =$	16.60		12.30		1.80	
$\eta =$	.01911		.01887		.01955	

TABLE XXVII.

HYSTERESIS OF TOOL-STEEL *S*.

	(1)		(2)		(3)		(4)	
F	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r
112	±14.55		+14.55		+14.55		+14.55	
100	14.45	14.31	14.45	14.32	14.48	14.42	14.51	14.49
90	14.35	14.09	14.35	14.10	14.40	14.28	14.43	14.38
80	14,25	13.74	14.25	13.76	14.29	14.12	14.34	14.25
70	14.09	13.28	14.09	13.31	14.12	13.88	14.20	14.07
60	13.87	12.74	13.87	12.77	13.83	13.58	13.97	13.85
50	13.57	11.94	13.57	11.99	13.46	13.20	13.63	13.55
40	13.08	10.70	13.08	10.89	13.02	12.78	+13.28	
30	12.32	8.60	12.35	9.55	12.41	12.30	[$F_2 = +43$]	
20	11.10	5.06	11.30	7.20	+11.90			
10	9.55	−.80	9.90	3.60	[$F_2 = +24$]			
0	±6.40		7.70	−.90				
−10			4.60	−3.80				
−20			−1.70	−6.20				
−30			−7.20					
$H =$	66.74		41.22		1.43		.48	
$L =$	14.55		10,875		1.325		.635	
$\eta =$	.01457		.01434		.01444		.01434	

TABLE XXVIII.

HYSTERESIS OF TOOL-STEEL—RESULTS.

No.		F_1	F_2	$F = \frac{F_1-F_2}{2}$	L_1	L_2	$L = \frac{L_1-L_2}{2}$	H	η obs.	$10^5 \Delta\eta$	= %
Hh, Glass-hard.									Av. η = .07476		~.075
(1)	*a*	+275	−275	275	+ 5.93	− 5.93	5.93	82.04	.07533	− 57	− .8
(2)	*p*	+275	− 83	179	+ 5.94	− 3.76	4.85	59.04	.07480	− 4	− .1
(3)	*p*	+275	− 45	160	+ 5.95	= 0	2.975	26.52	.07342	+134	+1.8
(4)	*p*	+275	+ 30	122.5	+ 5.97	+ 4.66	.655	2.42	.07546	− 70	− .9+
(5)	*a*	+124	−124	124	+ 5.12	− 5.12	5.12	64.50	.07493	− 17	− .2
(6)	*p*	+124	− 41	82.5	+ 5.13	= 0	2.565	21.46	.07533	− 57	− .8
(7)	*p*	+124	+ 15	54.5	+ 5.17	+ 3.88	.645	2.36	.07560	+ 16	+ .2+
H, Glass-hard, {after application of alternate current.}									Av. η = .06130		~.061
(1)	*a*	+120	−120	120	+ 6.25	− 5.80	6.025	68.52	.06136	− 6	− .1
(2)	*p*	+120	− 47	83.5	+ 6.25	− .12	3.185	24.98	.06124	+ 6	+ .1
(3)	*p*	+120	+ 28	46	+ 6.25	+ 4.95	.65	1.96	.06188	− 58	− .9+
(4)	*p*	− 28	−120	46	− 4.57	− 5.90	.665	2.03	.06178	− 48	− .8+
(5)	*p*	+120	+ 47	36.5	+ 6.25	+ 5.25	.50	1.29	.06197	− 67	−1.1+
(6)	*p*	− 47	−120	36.5	− 4.93	− 5.90	.485	1.21	.06103	+ 27	+ .4+
Oh, Oil-hardened									Av. η = .02670		~.027
(1)	*a*	+260	− 20	260	+13.25	−13.25	13.25	106.20	.02695	+ 25	+ .9
(2)	*p*	+260	− 30	145	+13.25	− 2.24	7.745	44.78	.02683	− 13	− .5
(3)	*p*	+260	+ 30	115	+13.25	+10.60	1.325	2.75	.02778	−108	−4.0+
(4)	*a*	+ 80	− 80	80	+11.30	−11.30	11.30	82.20	.02692	− 22	− .8
(5)	*p*	+ 80	− 26	53	+11.30	− .70	6.00	28.96	.02611	+ 59	+2.2
(6)	*p*	+ 80	+ 27	26.5	+11.30	+ 9.48	.91	1.48	.02727	− 57	−2.0+
O, Oil-hardened, {after application of alternate current.}									Av. η = .02700		~.027
(1)	*a*	+112	−112	112	+13.65	−13.65	13.65	111.64	.02700	0	
(2)	*p*	+112	+ 24	44	+13.65	+10.48	1.585	3.52	.02669	+ 31	+1.2+
(3)	*p*	+112	+ 43	34.5	+13.65	+11.95	.85	1.32	.02713	− 13	− .5+
Sh, Annealed.									Av. η .01899		~.019
(1)	*a*	+240	−240	240	+16.60	−16.60	16.60	108.00	.01911	− 12	− .6
(2)	*p*	+240	− 26	133	+16.60	− 8.00	12.30	66.00	.01887	+ 12	+ .6
(3)	*p*	+240	+ 26	107	+15.60	+13.00	1.80	3.16	.01955	+ 56	−3.0+
S, Annealed, {after application of alternate current.}									Av. η = .014455		~.0145
(1)	*a*	+112	−112	112	+14.55	−14.55	14.55	66.74	.01457	−11.5	− .8
(2)	*p*	+112	− 30	71	+14.55	− 7.20	10.875	41.22	.01434	+11.5	+ .8
(3)	*p*	+112	+ 24	44	+14.55	+11.90	1.325	1.43	.01444	+ 1.5	+ .1+
(4)	*p*	+112	+ 43	34.5	+14.55	+13.28	.635	.48	.01434	+11.5	+ .8+

F_1 and F_2 = maximum values of M. M. F. in ampere-turns per cm.
L_1 and L_2 = maximum values of metallic induction in kilolines per cm.^{2}
$F = \frac{F_1 - F_2}{2}$ and $L = \frac{L_1 - L_2}{2}$ are the amplitudes of the variation of M. M. F. and induction.
H = observed value of the dissipation of energy in kilo-ergs per cycle and cm.^{3}
η = coefficient of hysteresis calculated therefrom, and

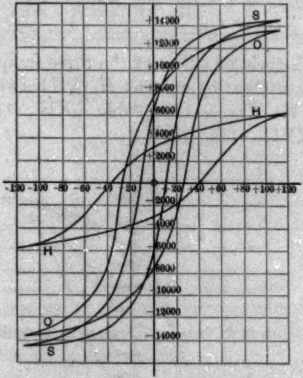


FIG. 6.—Welded Steel. Hysteretic Cycles.

$\triangle \eta$, the difference between the individual values and the average value of η, where again the cycles of small amplitude and therefore of lesser exactness are excluded in calculating the average of η. (The values *not* used for calculating av. η are marked by crosses +, as in the former tests.)

Again, we find the hysteretic loss dependent only upon the amplitude of the magnetic variation, but not upon their absolute values, and derive as constants of the six samples,

$$\rho = \alpha + \sigma F,$$

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6},$$

the values given in Table XXIX.

TABLE XXIX.

MAGNETIC CONSTANTS OF TOOL-STEEL.

	$F \geqq$	Coefficient of Magnetic Hardness α	Coefficient of Magnetic Saturation σ	Coefficient of Magnetic Hysteresis η	Absolute Saturation $L_\infty = \frac{1}{\sigma}$
Hh..............	90	8.0	.121	.0748	8.28
H...............	90	7.8	.105	.0613	9.53
Oh..	70	1.9	.066	.0267	15.16
O....	60	1.54	.060	.0270	16.70
Sh..............	60	1.33	.060	.0190	16.70
S...............	40	1.22	.0575	.0145	17.40

Fig. 6 gives a cycle of either of the three samples after the application of the alternating current *H*, *O*, *S* between the opposite and equal M. M. F's. $F = \pm 112$ [Table XXII., (1); Table XXV., (1); Table XXVII., (1)].

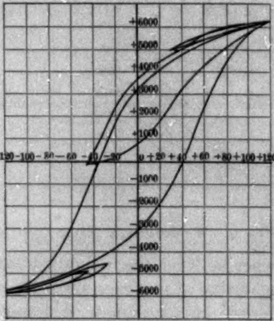


FIG. 7.—Glass-hard Steel. Hysteretic Cycles.

Fig. 7 gives the six magnetic cycles of *H* represented in Table XXII.

III. CAST-STEEL.

In the same manner as in Test II., two pieces of annealed cast-steel were treated.

Two pieces of annealed cast-steel were obtained from the same manufacturer, of the same casting, turned off to standard size, 20 cm. long and 4 cm.^{2} cross-section, and by comparing them on the magnetometer, found to be exactly alike. Then one was left annealed, the other heated and hardened in cold water. Although cast-steel, it was after this found mechanically very much harder. In Table XXX. are given the magnetic characteristics of both samples, annealed and hardened.

TABLE XXX.

MAGNETIC CHARACTERISTICS OF CAST-STEEL.

F	Annealed. ρ	Annealed. L	Hardened. ρ	Hardened. L
10	2.80	3.57		
15	2.23	6.70		
20	2.16	9.30	5.20	3.85
25	2.29	10.90	4.60	5.43
30	$\rho = .88 + .054\ F$	12.00	4.50	6.67
40		13.15	$\rho = 2.7 + .054\ F$	8.24
60		14.60		10.20
80		15.40		11.40
100		15.90		12.35
150		16.73		13.90
[200		17.10		14.82
300		17.55		15.88
400		17.84		16.50
500		17,95		16.88]
Absolute saturation		18.50		18.50

As seen, for low M. M. F's. the two samples are magnetically very different, but approach each other for higher M. M. F's. and reach the same value of saturation.

TABLE XXXI.

HYSTERESIS OF HARDENED CAST-STEEL.

F	(1) L_d	(1) L_r	(2) L_d	(2) L_r	(3) L_d	(3) L_r	(4) L_d	(4) L_r	(5) L_d	(5) L_r
+82	±11.58		+11.58		+11.58		+11.58		+11.58	
70	11.35	10.94	11.32	10.87	11.28	10.98	11.29	11.14	11.35	11.21
60	11.00	10.20	11.02	10.12	10.92	10.34	10.96	10.62	11.04	10.75
50	10.57	9.12	10.63	9.18	10.50	9.60	10.53	9.89	10.60	10.23
40	10.06	7.05	10.13	7.72	10.00	8.70	10.08	9.37	10.33	10.06
30	9.51	3.40	9.62	5.05	9.47	7.65	9.69	9.00	10.13	10.05
20	8.90	−1.80	9.03	−.10	8.92	5.80	9.32	8.72	+10.05	
10	8.20	−5.70	8.32	−4.35	8.28	.80	8.93	8.49	[$F_2 = +27.5$]	
0	±7.33		7.40	−5.93	7.60	−.30	+8.42			
−10			5.70	−6.80	6.30	−.70	[$F_2 = 0$]			
−20			1.50	−7.52	1.25	−.82				
−30			−3.65	−8.06	−.81					
−40			−6.90	−8.53	[$F_2 = -26.5$]					
−53			−9.07							
$H =$	87.63		72.905		32.51		3.645		1.14	
$L =$	11.58		10.325		6.195		1.58		.765	
$\eta =$	.02760		.02758		.02784		.02779		.02770	

TABLE XXXII.

HYSTERESIS OF HARDENED CAST-STEEL.

F	(6) L_d	(6) L_r	(7) L_d	(7) L_r	(8) L_d	(8) L_r	(9) L_d	(9) L_r	(10) L_d	(10) L_r	(11) L_d	(11) L_r
+45.6	±8.70		+8.75		+8.96		+8.96		+8.96		+8.96	
40	8.50	7.77	8.57	8.07	8.76	8.30	8.76	8.30	8.76	8.34	8.76	8.41
35	8.28	6.35	8.38	7.31	8.51	7.70	8.51	7.70	8.53	8.02	8.53	8.22
30	8.03	4.51	8.16	6.35	8.25	7.00	8.25	7.10	8.27	7.78	8.33	8.20
25	7.77	2.42	7.92	5.03	7.97	6.11	7.96	6.27	8.02	7.59	+8.20	
20	7.46	— .33	7.63	2.70	7.63	4.86	7.66	5.28	7.73	7.48	[F_2 = + 27]	
15	7.07	—2.53	7.29	— .55	7.31	2.75	7.33	4.18	7.57	7.42		
10	6.63	—3.88	6.85	—2.12	6.87	.66	6.93	3.19	+7.42			
+ 5	6.12	—4.82	6.38	—2.90	6.43	.22	6.50	2.31	[F_2 = + 13.5]			
0	±5.54		5.83	—3.44	5.90	. 0	6.05	1.65				
— 5			5.26	—3.90	5.28	—.17	5.44	1.25				
—10			4.54	—4.28	4.51	—.33	4.73	1.00				
—15			3.19	—4.61	3.30	—.48	3.63	.92				
—20			—1.00	—4.90	.30	—.62	+.92					
—25			—3.90	—5.10	—.70		[F_2 = — 18.5]					
—31.6			—5.31		[F_2 = — 22]							
H =	56.11		38.72		22.42		16.10		1.10		.38	
L =	8.70		7.03		4.83		4.02		.77		.38	
η =	.02792		.02836		.02859		.02754		.02649		.02832	

TABLE XXXIII.

HYSTERESIS OF CAST-STEEL—RESULTS.

No.		F_1	F_2	$F = \frac{F_1 - F_2}{2}$	L_1	L_2	$L = \frac{L_1 - L_2}{2}$	H	η obs.	$10^5 \triangle \eta$	= %
Hardened.......................... Av. η = .02792											~.028
(1)	*a*	+82	—82	82	+11.58	—11.58	11.58	87.63	.02760	+ 32	+1.1
(2)	*p*	+82	—53	67.5	+11.58	— 9.07	10.325	72.905	.02758	+ 34	+1.2
(3)	*p*	+82	—26.5	54.2	+11.58	— .81	6.195	32.51	.02784	+ 8	+ .3
(4)	*p*	+82	= 0	41	+11.58	+ 8.42	1.58	3.645	.02779	+ 13	+ .5+
(5)	*p*	+82	+27.5	27.2	+11.58	+10.05	.765	1.14	.02770	+ 22	+ .7+
(6)	*a*	+45.4	—45.4	45.4	+ 8.70	— 8.70	8.70	56.11	.02792	0	0
(7)	*p*	+45.4	—31.6	38.5	+ 8.75	— 5.31	7.03	38.72	.02836	— 44	—1.6
(8)	*p*	+45.8	—22	33.9	+ 8.96	— .70	4.83	22.42	.02859	— 67	—2.4
(9)	*p*	+45.8	—18.5	32.1	+ 8.96	+ .92	4.02	16.10	.02754	+ 38	+1.4
(10)	*p*	+45.8	+13.5	16.1	+ 8.96	+ 7.42	.77	1.10	.02649	+143	+5.1+
(11)	*p*	+45.8	+27	9.4	+ 8.96	+ 8.20	.38	.38	.02832	— 40	—1.4+
Annealed.......................... Av. η = .008481											~.0085
(1)	*a*	+100	—100	100	+15.85	—15.85	15.85	35.00	.008502	—2.1	— .4
(2)	*a*	+ 44	— 44	44	+13.62	—13.62	13.62	44.40	.008460	+2.1	+ .4

Tables XXXI. and XXXII. give a number of cycles made with the hardened piece *h*, and Table XXXIII. the results of these cycles and of two cycles made with the annealed piece, the denotation being the same as before.

Herefrom we derive the results for this cast-steel,

$$\rho = a + \sigma F,$$

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6},$$

		Coefficient of		
		Magnetic Hardness.	Magnetic Saturation.	Magnetic Hysteresis.
	$F \geqq$	a	σ	η
Soft cast-steel *s*,	30	.88	.054	.00848
Hardened cast-steel *h*,	40	2.7	.054	.02792

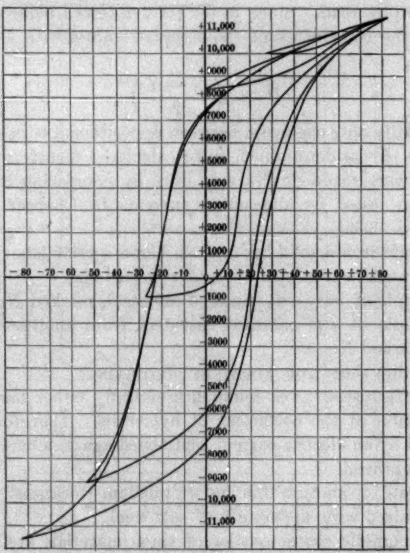


FIG. 8.—Hard Cast-Steel. Hysteretic Cycles.

The magnetic characteristics of these two samples of cast-steel, together with many other characteristics, are represented in Figs. 17 and 21. Fig. 8 gives the five cycles of hardened cast-steel from Table XXXI.

Numerous data on the magnetic constants of different kinds of cast-steel are given in Chapter III. and collected in tables XLVII. and LI, represented in Figs. 16, 17 and 21.

IV. DIFFERENT KINDS OF IRON AND STEEL.

A number of tests were made with different kinds of iron and soft steel, to determine the magnetic constants a, σ, η.

Here the differential method was used for the determination of the coefficient of magnetic hysteresis η, that is the test piece was balanced step by step against a sample of known magnetic hysteresis, usually Norway iron or the sheet iron of Chapter I. and so the difference in the dissipation of energy by hysteresis in both samples read. Since in the former tests I believe to have proved the coincidence of the observed values with the general formula,

$$H = \eta\left(\frac{L_1 - L_2}{2}\right)^{1.6},$$

here usually only one cycle, between opposite and equal values of M. M. F. F was determined, and η calculated therefrom.

Tests were made on Norway iron, by comparing it with the sheet-iron tested by alternating currents in Chapter I., which gave $\eta = .0035$.

Wrought-iron, a solid bar of 4 cm.^{2} cross-section (standard size).

Mitis metal, cylindrical piece of standard size.

A sample of very soft annealed cast-steel, marked No. 6.

A sample of soft annealed cast-steel, from another manufacturer, marked No. 5.

Very thin sheet-iron, known as "ferrotype."

This "ferrotype" was found magnetically rather hard, and of a high value of the coefficient of hysteresis. Therefore it was annealed by an electric current and tested again, whereby it was found improved.

Tin plate, 2 samples, thin and of medium thickness.

Galvanized wire, apparently of soft steel.

The magnetic characteristics of these materials are given in Table XXXIV., and to a great part shown as curves in Fig. 17.

TABLE XXXIV.

Different Kinds of Iron and Steel.

	Norway Iron, Standard.		Wrought-Iron in Bars.		Very Soft Annealed Cast-Steel No. 6.		Soft Annealed Cast-Steel No. 5.		Mitis Metal.		Ferrotype, Commercial.		Ferrotype, Annealed.		Tin Plate, Thin.		Tin Plate, Medium Thickness.		Soft Galvanized Steel Wire.	
F	ρ	L	ρ	L	ρ	L	ρ	L	ρ	L	ρ	L	ρ	L	ρ	L	ρ	L	ρ	L
5.5	$\rho = .166 + .05435\,F$	11.83	.54	10.20	$\rho = .232 + .05577\,F$		$\rho = .260 + .05511\,F$		.77	7.15	1.10	5.00								
7.5		13.08	.64	10.96		11.50			.79	8.88	1.01	7.44	1.05	7.15	.75	10.00	.98	7.65	1.79	4.20
8.5		13.56	.70	12.11					.81	10.54										
10		14.10	.77	13.06		12.66		12.33	.88	11.43	1.04	9.63	.97	10.32	.78	12.85	.98	10.20	1.71	5.85
11.5		14.56	.85	13.57					.92	12.50										
12.5		14.80	$\rho = .20 + .05547\,F$	13.98		13.45		13.12	$\rho = .30 + .05444\,F$	12.75	1.10	11.38	1.03	12.15	$\rho = .192 + .05464\,F$	14.28	1.01	12.38	1.70	7.37
15		15.30		14.55		14.03		13.80		13.44	1.20	12.50	1.12	13.40		14.80	$\rho = .321 + .05315\,F$	13.44	1.75	8.60
20		16.00		15.30		14.84		14.68		14.40	$\rho = .450 + .04975\,F$	13.75	$\rho = .337 + .04905\,F$	14.93		15.60		14.45	$\rho = .67 + .066\,F$	10.05
25		16.42		15.80		15.40		15.30		15.07		14.80		15.85		16.08		15.15		10.80
30		16.72		16.10		15.75		15.68		15.55		15.45		16.40		16.40		15.68		11.30
40		17.10		16.60		16.28		16.28		16.18		16.40		17.23		16.85		16.30		12.10
60		17.53		17.03		16.78		16.85		16.85		17.48		18.03		17.30		17.12		12.94
80		17.74		17.30		17.08		17.10		17.20		18.00		18.55		17.55		17.50		13.44
100		17.83		17.42		17.25		17.55		17.45		18.40		18.80		17.68		17.75		13.80
200		18.15		17.72		17.60		17.73		17.90		19.30		19.45		18.00		18.30		14.50
Absolute Saturation		18.40		18.03		17.95		18.15		18.37		20.10		20.10		18.30		18.81		15.15
Coefficient of hysteresis η =		.002275		.003260		.003181		.004573		.004281		.00548		.00458		.002863		.004255		.00349

The results of the tests, without exception proved the law of metallic magnetic reluctively,

$$\rho = \alpha + \sigma F.$$

The results are,

(1.) *Norway Iron.*

This is the softest metal magnetically and has the lowest coefficient of hysteresis I ever observed, little larger than the "soft iron wire" of Ewing. It is the piece used as Standard in the Differential Magnetometer. The whole instrument is built of this material.

The dissipation of energy by hysteresis, and the other magnetic constants were found,

$\pm F$	$\pm L$	H	η	α	σ	L_∞
75	17.70	14.25	.002275	.166	.05435	18.40

for $F \geqq 5$

(2.) *Ordinary Good Wrought-Iron in Bars.*

The hysteresis and the other magnetic constants are,

$\pm F$	$\pm L$	H	η	α	σ	L_∞
75	17.20	19.50	.003260	.20	.05547	18.03

for $F \geqq 12$

(3.) *Mitis Metal.*

The hysteresis and the other magnetic constants are,

$\pm F$	$\pm L$	H	η	α	σ	L_∞
75	17.11	25.40	.004281	.30	.05444	18.37

for $F \geqq 12$

As seen, magnetically this mitis metal behaves almost exactly like wrought-iron and sheet-iron. Its coefficient of magnetic hardness is $\alpha = .30$, while for different kinds of sheet-iron and wrought-iron I found values varying between .166 (Norway iron) and .35 (thick sheet-iron), and in unannealed ferrotype even .45. The coefficient of magnetic saturation $\sigma = .05444$ is about the average found for different samples of wrought-iron, which vary between .058 (the sample of sheet-iron, given in Chapter I.) and .04975 (ferrotype), while Norway iron has $\sigma = .05435$, that is almost the same as mitis metal.

The coefficient of hysteresis $\eta = .00428$ is somewhat larger, but still within the limits of sheet-iron, which reaches .0045 in a sample described on p. 26 in my former paper and was found still higher in ferrotype. Hence, the conclusion to be derived herefrom is, "*For all practical purposes mitis metal is to be considered magnetically as identical with ordinary good wrought-iron.*"

(4.) *Very Soft Annealed Cast-Steel*, No. 6.

The hysteresis and the other magnetic constants are,

$\pm F$	$\pm L$	H	η	a	σ	L_∞
75	17.00	18.67	.003181	.232	.05567	17.95

for $F \geqq 6$

As seen, this annealed cast-steel is *far superior to ordinary good wrought-iron*, and almost approaches Norway iron.

The magnetic hardness $a = .232$ is about midway between that of Norway iron, and the lowest value found in ordinary good sheet-iron.

The coefficient of magnetic saturation is about the same as that of wrought-iron and sheet-iron.

The coefficient of magnetic hysteresis is lower than for average wrought-iron.

(5.) *Soft Annealed Steel*, No. 5.

The hysteresis, and the magnetic constants are,

$\pm F$	$\pm L$	H	η	a	σ	L_∞
75	17.00	26.84	.004573	.260	.05511	18.15

for $F \geqq 10$

Even this annealed cast-steel is in its magnetic hardness $a = .260$ still superior to average wrought-iron, in magnetic saturation equal, and with its coefficient of hysteresis, still in the range of wrought-iron. Both the materials, Nos. 5 and 6, are used for the magnetic field in the Eickemeyer-Field street car motors.

(6.) *Ferrotype.*

Twenty-three strips of 20 cm. length, 1.27 cm. width and .015 cm. thickness (calculated from weight, by specific gravity 7.7), that is of .019 cm.^{2} cross-section, were used, giving a joint cross-section of .438 cm.^{2} This material is remarkable in so far as it reaches a very high value of magnetic saturation, over 20,000 lines of magnetic force per cm.^{2} But with regard to magnetic hardness and hysteresis it was found poor; perhaps it was rolled rather cold, and thereby hardened. Hence, after testing it once, I annealed it. Each strip was fastened with its ends between two clamps, and a (continuous) current of about $50 \sim 60$ amperes sent through, which heated it to bright red. The current was applied repeatedly. About 10 per cent were burnt off, leaving a joint cross-section of .396 cm.^{2}

The hysteresis and the magnetic constants are,

	$\pm F$	$\pm L$	H	η	a	σ	L_∞
not annealed:	65	17.6	34.04	.00548	.45	.04975	20.10
annealed:	65	18.2	30.00	.00458	.337		

$F \geqq 15 \sim 20.$

As already stated, this material is remarkable for its high magnetic saturation.

(7.) *Tin-Plate.*

Two samples of ordinary commercial tin-plate were tested, of the thickness .0268 cm. and .0378 cm. (calculated from weight and including the tin.) The length of the test pieces was 20 cm., the width 2.55 cm.

Of the thicker sample 22 pieces were used, of a joint cross-section of 2.12 cm.^{2}, of the thinner sample 30 pieces were used, of 2.05 cm.^{2} joint cross-section. Considerable difference was found between the two samples, while the thicker sample equalled ordinary and even rather poor sheet-iron, the thinner sample was superior to any sheet-iron, and came very near to Norway iron.

The hysteresis and the magnetic constants are,

Thicker sample .0378 cm. thick.

$\pm F$	$\pm L$	H	η	a	σ	L_∞	$F \geqq$
26	15.31	21.0	.004229				
62	17.15	25.5	.004282				
		av.	.004255	.321	.05315	18.81	14

Thinner sample .0268 cm. thick.

$\pm F$	$\pm L$	H	η	a	σ	L_∞	$F \geqq$
26	16.[illegible]3	15.4	.002853				
62	17.33	17.4	.002873				
		av.	.002863	.192	.05464	18.30	12

In these values no reduction has been made for the tin-covering of the sheet-iron, but these figures refer to the whole cross-section of the tin-plate, including the tin. Therefore, especially in the thinner sample, in the iron proper L_∞ will be a little higher than given.

(8.) *Galvanized Iron (Steel?) Wire.*

One hundred and forty-three pieces of wire, of 20 cm, length and .0193 cm.^{2} cross-section (calculated from weight, specific gravity 7.7), that is of .157 cm. diameter, were used, giving a joint cross-section of 2.76 cm.^{2}

The hysteresis and the magnetic constants are,

$\pm F$	$\pm L$	H	η	a	σ	L_∞.
80	13.35	13.78	.003455			
32	11.50	10.85	.003454			
18	9.70	8.50	.003550			
	av.	$\eta =$	.00349	.67	.066	15.15
			$\sim$ .0035	for $F \geqq 20$		

As seen, the constants σ and a have values found in soft cast-steel, but η is remarkably low, in the range of average wrought-iron.

V. Amalgam of Iron.

In the amalgams of iron we have a very interesting class of alloys in-so-far as they bridge over the wide gap existing between the paramagnetic materials, as iron, nickel, cobalt, etc., and the non-magnetic materials, as air, etc. It is not easy to get amalgam of iron, since iron does not dissolve in mercury, and is not even wetted thereby. But when separated in molecular form, iron dissolves readily. So by electrolyzing a solution of ferro-sulphate $S\,O_4\,Fe$ with mercury as cathode by a dense electric current, the iron, deposited in molecular form, dissolved in quicksilver; and by pressing the quicksilver through a piece of linen, a solid, crystalline amalgam was separated from a liquid one. This liquid amalgam still contained a certain amount of iron in solution, as its attraction by the magnetic-pole showed; but was not sufficiently magnetizable to make tests with it.

With great current density and small supply of mercury, sometimes a crystallized amalgam of dark steel color was separated, in needle-formed crystallization. This amalgam evidently contained still more iron, but was not tested.

The crystalline amalgam, which was still pliable enough to be pressed into a solid body, contained 11 per cent. of iron, and small traces of foreign matter, as a chemical analysis showed. Since it evidently still contained traces of the liquid amalgam, it may about correspond to the formula,

$$Fe\,Hg_2$$

All these amalgams were liable to slow decomposition, and separated in a few weeks a part of the iron as fine black powder. Hence they had to be tested soon after preparation. It was placed in a fibre tube and compressed by two wrought-iron pieces which from either side screwed into the tube, thereby affording a path for the magnetism. These Norway iron cylinders were balanced by an equal pair of cylinders at the other side of the instrument, and the amalgam tested then.

The dimensions of the tested piece of amalgam were,

Length, 4 cm.
Cross-section, 4.45 cm.^{2}, cylinder.

Although showing strong attraction against a magnet-pole, the amalgam had only about twice the permeability of air.

Table XXXV. gives the magnetic characteristic of the amalgam containing 11 per cent. of iron, with the usual denotation, $L =$ metallic induction, $\rho =$ metallic reluctivity.

TABLE XXXV.

MAGNETIC CHARACTERISTIC OF AMALGAM OF IRON, 11 %.

F	L	ρ obs.	ρ calc.
20	22	909	Approximately, $\rho = 500 + 1.12\,F$ for $F \geqq 240$
40	49	816	
60	76	790	
80	103	775	
100	130	769	
120	157	764	
140	184	761	
160	211	759	
180	238	756	
200	265	755	
220	290	759	
240	310	774	
260	328	792	
280	345	811	
300	360	833	
320	374	856	
340	387	879	
360	399	902	
[Absolute Saturation	900]		

For higher values of M. M. F., $F \geqq 240$, the metallic reluctivity can *approximately* be expressed by the equation,

$$\rho = 500 + 1.12\,F$$

though the bend in the curve is so small, that the constants α and σ are rather uncertain.

Table XXXVI. gives a cycle of hysteresis,

TABLE XXXVI.

HYSTERESIS OF AMALGAM OF IRON, 11 %.

F	L_d	L_r
320	± .375	
250	.326	.308
200	.285	.252
150	.238	.185
100	.182	.112
50	.118	.033
0	± .045	
$H =$	3.04	
$L =$	.375	
$\eta =$	.2314	

The results are,

	$\pm F$	$\pm L$	H	η	a	σ	L_∞	Coercitive Force.
Amalgams of iron,	320	.375	3.04	.2314	500	1.12	.900	$F = 28$.
Common air,	320	.400	0	0	800	0	∞	

All the three coefficients, η, a, σ, are unusually high in this material, the "absolute saturation" amounting to only,

$$L_\infty = 900.$$

Fig. 9 gives the magnetic characteristic and one cycle of hysteresis of this amalgam of iron. The dotted straight line denotes the magnetic characteristic of air, H which has to be added to get the whole induction, $B = L + H$.

Since 11 per cent. of weight corresponds to about 17.5 volume per cent., the magnetic constants referred to the volume of iron contained in the amalgam are,

η	a	σ	L_∞
.0815	87.5	.196	5.10

η is still higher than the highest values found for glass-hard steel. (*cf.* Chapter V.)

In the same manner as amalgam of iron, *amalgam of nickel* was prepared by electrolysis, and gave the three amalgams:

1. A liquid amalgam, consisting of quicksilver with traces of nickel, but showing no perceptible influence upon the magnet-needle.

2. A silver-colored, pliable amalgam, containing apparently about 10 per cent. of nickel. This amalgam seems to be entirely non-magnetic, since I could get no deflection of the compass-

needle by it. It dissociates very rapidly, even when dry. By heating in boiling paraffin, or at ordinary temperature within a day, it was always found dissociated into quicksilver (or the first amalgam) and the third amalgam.

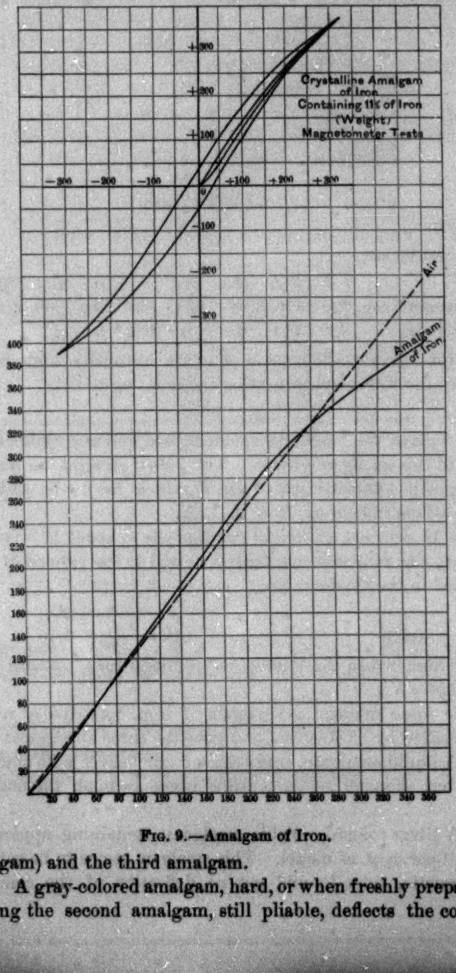


FIG. 9.—Amalgam of Iron.

3. A gray-colored amalgam, hard, or when freshly prepared by heating the second amalgam, still pliable, deflects the compass-

needle strongly, and becomes permanently (and relatively strongly) magnetized in the magnetic field. Though an allotropic modification of this amalgam seems to exist, which is *unmagnetic*. No exact tests have yet been made with these amalgams.

VI. Porous Iron.

By heating this amalgam of iron to dull red heat, the mercury evaporated, and a very porous mass of iron, containing some percentage of oxides, remained. The material contracted considerably hereby, from 14.75 cm.^{3} to 8.055 cm.^{3}, but was, nevertheless, full of smaller and larger pores, containing very nearly 30 volume percentage of iron.

TABLE XXXVII.

MAGNETIC CHARACTERISTICS OF POROUS IRON, 30 VOLUME PER CENT.

F	(1) L	ρ	(2) L	ρ
20	.53	$\rho = 25.4 + .604\,F$	.22	$\rho = 76 + .71\,F$
40	.81		.38	
60	.97		.50	
80	1.08		.60	
100	1.16		.68	
120	1.23		.75	
150	1.30		.82	
200	1.37		.92	
300	1.45		1.04	
[500	1.53		1.16	]
Absolute saturation........	1.66		1.41	

TABLE XXXVIII.

HYSTERESIS OF POROUS IRON, 30 VOLUME PER CENT.

F	L_d	L_r
140	± 1.28	
130	1.26	1.26
120	1.23	1.23
110	1.20	1.19
100	1.17	1.15
90	1.13	1.11
80	1.09	1.06
70	1.04	1.00
60	.98	.93
50	.92	.84
40	.86	.73
30	.78	.59
20	.69	.35
10	.59	—.06
0	± .43	
$H =$	3.98	
$L =$	1.28	
$\eta =$	.0425	

The test piece had the following dimensions,

Length, 4.45 cm.
Cross-section, 1.81 cm.^{2}, almost square.
Volume, 8.055 cm.^{3}.

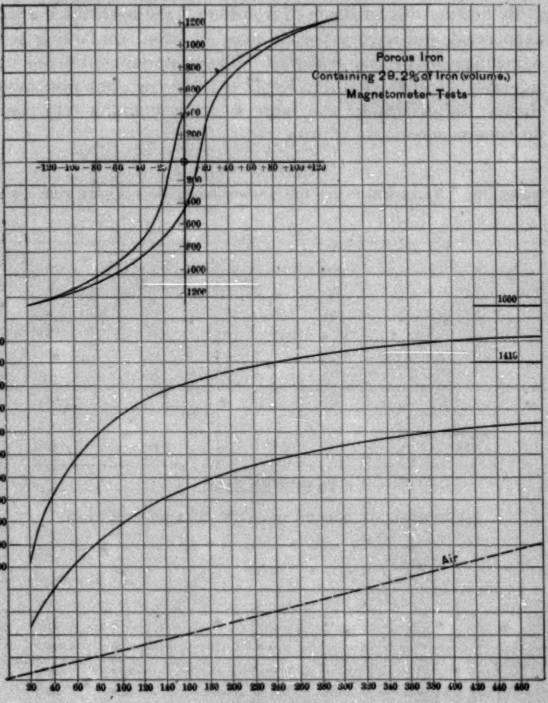


FIG. 10.—Porous Iron.

Its magnetic characteristic is given in Table XXXVII., in column 1, a cycle of hysteresis in Table XXXVIII.

The results are,

$\pm F$	$\pm L$	H	η	a	σ	L_∞
140	1.28	3.98	.0425	25.4	.604	1.66

$F \geqq 90.$

Another piece of such porous iron, of the dimensions,

Length, 6.03 cm.; cross-section, .53 cm.^{2}; volume, 3.2 cm.^{3}; containing 31 volume per cent. of solid matter, but much impurer, gave the characteristic in Table XXXVII., Column 2, expressed by the equation,

$$\rho = 76 + .71\, F$$
$$\text{for } F \geqq 90.$$

Here again are noteworthy the high values of magnetic hardness and hysteresis, and the low value of magnetic saturation, $L_\infty = \frac{1}{\sigma}$, which lies at 1660 viz. 1410.

Fig. 10 gives the magnetic characteristics of both samples with the air-characteristic as dotted lines for comparison, and one cycle of hysteresis. It is noteworthy, that the hysteretic cycle is entirely unlike that of the iron-amalgam, where the porous iron was derived from, and resembles much more a cast-iron cycle, but of one-eighth the height of ordinates. The first sample was heated to dull red heat, for evaporating the mercury, the second one heated over the alcohol lamp, had not become as hot. This may account for its far greater magnetic hardness.

Referred to the volume of the iron contained in the test pieces, 30 and 31 per cent. respectively, their magnetic constants are,

	η	a	σ	L_∞
(1)	.0206	7.6	.181	5.52
(2)		23.6	.22	4.55

The value $\eta = .0206$ corresponds to that of medium hard steel, and so the test pieces behaved, getting strongly and permanently magnetized.

VII. MAGNETITE.

With a piece of magnetite (Magnetic Iron Ore) of 6 cm.^{2} cross-section (square) and 6.5 cm. length, a very pure sample, derived from the Tilly Foster Mines, Brewsters, Putnam County, State of New York, a large number of tests were made.

The magnetic characteristic is given in Table XXXIX.

TABLE XXXIX.

MAGNETIC CHARACTERISTIC OF MAGNETITE (MAGNETIC IRON ORE).

F	*L*	*ρ*
15	.71	21.0
20	1.09	18.4
25	1.47	17.0
30	1.80	16.7
35	2.07	16.9
40	2.28	17.5
45	2.43	
50	2.56	$\rho = 8.9 + .213\,F$
60	2.77	
80	3.08	
100	3.31	
120	3.48	
140	3.62	
160	3.72	
180	3.81	
200	3.89	
[300	4.12	
500	4.33]	
Absolute Saturation......	4.69	

TABLE XL.

HYSTERESIS OF MAGNETITE (MAGNETIC IRON ORE),

	(1)		(2)		(3)		(4)		(5)		(6)	
F	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r
0	±.60		+.80		±.89		+.94					
5	.92	−.20	1.16	−.36	1.24	−.46	1.28	−.55				
10	1.17	+.30	1.43	+.14	1.50	+.06	1.54	−.05	[+16]			
15	1.39	.80	1.68	.67	1.73	.58	1.76	+.44	+1.82			
20	1.55	1.20	1.87	1.10	1.90	1.02	1.96	.90	1.97	1.85		
25	1.68	1.52	2.04	1.46	2.10	1.39	2.14	1.30	2.15	1.99	[+30]	
30	±1.77		2.19	1.75	2.27	1.70	2.30	1.64	2.30	2.13	+2.32	
35	[±29]		2.32	2.00	2.40	1.95	2.43	1.89	2.43	2.24	2.44	2.34
40			2.43	2.20	2.53	2.16	2.55	2.10	2.55	2.36	2.56	2.45
45			2.53	2.37	2.64	2.34	2.66	2.29	2.65	2.48	2.65	2.54
50			2.61	2.52	2.72	2.47	2.75	2.43	2.74	2.58	2.74	2.64
55			2.68	2.66	2.81	2.59	2.84	2.55	2.82	2.69	2.82	2.72
60			±2.69		2.88	2.70	2.91	2.66	2.90	2.78	2.90	2.82
65			[±57]		2.94	2.80	2.98	2.77	2.96	2.87	2.96	2.89
70					3.00	2.90	3.04	2.87	3.01	2.95	3.01	2.96
75					3.06	3.00	3.10	2.97	3.06	3.02	3.06	3.03
80					3.11	3.08	3.15	3.05	3.11	3.09	3.11	3.10
85					3.15	3.14	3.20	3.12	3.15	3.14	3.15	3.15
90					±3.18		3.24	3.18	+3.18		+3.18	
95					[+88]		3.29	3.23	[+88]		[+88]	
100							3.33	3.28				
105							3.37	3.32				
110							3.41	3.37				
115							3.44	3.41				
120							3.48	3.45				
125							3.51	3.49				
130							3.55	3.53				
135							3.58	3.57				
140							±3.61					
$H =$	3.69		7.23		9.45		11.52		.81		.38	
$L =$	1.77		2.69		3.18		3.61		.68		.43	
$\eta =$	.02345		.02352		.02353		.02342		.02379		.02324	

Av. $\eta = .02348$.

Beyond the M. M. F. $F = 40$ the magnetic reluctivity strictly follows the linear law,

$$\rho = 8.9 + .2132\, F,$$

giving a characteristic similar to that of cast-iron, only that absolute saturation is already reached at the metallic induction,

$$L_\infty = 4.69.$$

To determine whether the law of the 1.6th power holds for the hysteretic loss of energy in magnetite also, a number of magnetic cycles were taken, which are given in Table XL., first between opposite and equal limits, $\pm F = 29, 57, 88, 140$ then between high values of induction of the same sign, between $F_1 = +88$ and $F_2 = 30$ and 16 respectively.

The results of these cycles are given in Table XLI.

TABLE XLI.

HYSTERESIS OF MAGNETITE (MAGNETIC IRON ORE)—RESULTS.

No.		F_1	F_2	$F = \frac{F_1 - F_2}{2}$	L_1	L_2	$L = \frac{L_1 - L_2}{2}$	H	η obs.	$10^5 \triangle \eta$	= %
(6)	*p*	+ 88	+ 30	29	+3.18	+2.32	.43	.38	.02324	+24	+1.0
(5)	*p*	+ 88	+ 16	36	+3.18	+1.82	.68	.81	.02379	−31	−1.3
(1)	*a*	+ 29	− 29	29	+1.77	−1.77	1.77	3.69	.02345	+ 3	+ .1
(2)	*a*	+ 57	− 57	57	+2.69	−2.69	2.69	7.23	.02352	− 4	− .2
(3)	*a*	+ 88	− 88	88	+3.18	−3.18	3.18	9.45	.02353	− 5	− .2
(4)	*a*	+140	−140	140	+3.61	−3.61	3.61	11.52	.02342	+ 6	+ .3
							Av. η =		.02348		

They prove conclusively, that the same law of hysteresis holds for magnetite.

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6}$$

and give as magnetic constants of *magnetite*,

η	a	σ	L_∞	$F \geq$
.02348	8.9	.2132	4.69	40

Fig. 11 gives the cycles of Table XL., 1, 2, 3 and 4, made between oppositely equal limits.

The two tests made on another sample and published in the paper of January 19th, 1892, give, $\eta = .020$, that is nearly the same

VIII. Ewing's Tests.

Before leaving the consideration of the phenomenon of hysteresis in *iron* and its alloys and compounds, I may be allowed to dwell upon some determinations of the loss of energy by hysteresis, made by Ewing, and given in his book on "Magnetic Induction in Iron and other Metals."

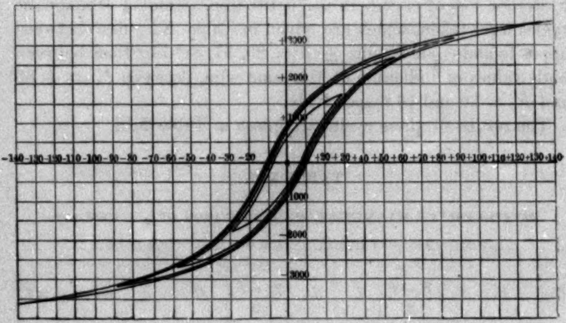


Fig. 11.—Magnetite. Hysteretic Cycles.

TABLE XLII.

MAGNETIC CYCLES OF SOFT IRON WIRE.

(Ewing, p. 106.)

F	*L*	*H* obs.	*H* calc.	*H* — *H* calc. obs.	= %
1.20	1.974	.41	.375	+.035	+8.5
1.56	3.83	1.16	1.082	+.058	+5.0
2.05	5.95	2.19	2.190	——	——
2.41	7.18	2.94	2.956	—.016	— .5
3.01	8.79	3.99	4.08	—.090	—2.3
3.97	10.59	5.56	5.51	+.050	+ .9
5.30	11.47	6.16	6.26	—.100	—1.7
5.63	11.95	6.59	6.69	—.100	—1.5
21.2	13.69	8.69	8.31	+.380	+4.4
60.2	15.48	10.04	10.11	—.070	— .7
Av. η = .002				±.090	±2.5

TABLE XLIII.

MAGNETIC CYCLES OF ANNEALED PIANOFORTE STEEL WIRE.

(Ewing, p. 109.)

F_1	F_2	$F = \frac{F_1 - F_2}{2}$	L_1	L_2	$L = \frac{L_1 - L_2}{2}$	H obs.	H calc.	$H - H$ calc. obs.	= %
+ 8	− 8	8	+ 1.51	− .94	1.225	1.20	1.52	+ .32	+21
+12	−10.4	11.2	+ 3.64	− 2.32	2.98	5.50	6.32	+ .82	+13
+15.2	−15.2	15.2	+ 5.66	− 4.90	5.28	15.90	15.80	− .10	− .6
+18.4	−19.2	18.8	+ 7.53	− 7.43	7.48	27.30	27.50	+ .20	+ .8
+24	−24	24	+ 9.45	− 9.55	9.50	41.90	40.20	−1.70	− 4.2
+65	−65	65 (?)	+13.80	−13.80	13.80	71.80	73.50	+1.70	+ 2.2

Av. $\eta = .01742$.

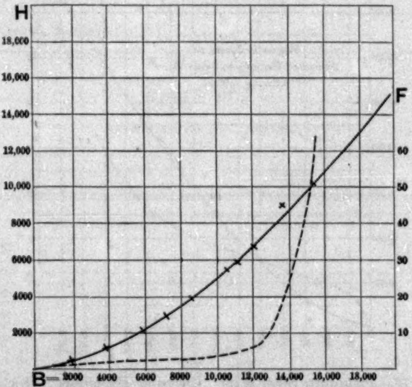


Fig. 12.—Soft Iron. Curve of Hysteresis. [Ewing.]

In Table XLII. and Fig. 12 are given the results of the graded cycles of hysteresis of very soft iron wire (pages 106–7 Ewing).

In Table XIII. and Fig. 5 are given the results of the graded cycles of hysteresis of medium good cast-iron, (No. 1).

In Table XLIII. and Fig. 13 are given the results of the graded cycles of annealed pianoforte steel wire (page 109 Ewing). These latter are taken from the plotted curve published by Ewing; hence only a considerable lesser exactness can be expected since the numerical data are not published by Ewing, as far as I know, and printed curves are never very exact, and not improved by measuring.

The data in Table XLII. are of interest in so far as they are the lowest values of hysteretic loss ever observed on iron. so far as I know. From these figures I found the law of the 1.6th power, two years ago, when trying to find a misprint which got into the table of the hysteretic loss, given in Kapp's "Alternate-Current Machinery" and calculated from these tests.

The denotations are the same as before,

F_1 and F_2 = the highest and the lowest values of E. M. F., in ampere turns per cm.

B_1 and B_2 = the highest and the lowest value of magnetic induction, in kilolines per cm.^{2}.

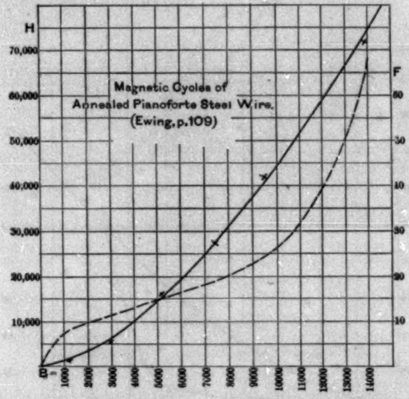


FIG. 13.—Pianoforte Steel Wire. Curve of Hysteresis. [Ewing.]

$F = \frac{F_1 - F_2}{2}$ and $B = \frac{B_1 - B_2}{2}$ = their amplitudes, or half their variations.

H = the energy consumed by hysteresis, during one complete cycle, in kiloergs per cm.^{3}.

η = coefficient of hysteresis, calculated therefrom.

Two further cycles, with annealed and with glass-hard pianoforte steel wire (Ewing page 84) give the results,

	F_1	F_2	$\frac{F_1-F_2}{2}$	B_1	B_2	$\frac{B_1-B_2}{2}$	H	η
Annealed pianoforte wire.	+75	−74	74.5	+14.2	−14.4	14.3	95.46	.022
Glass hard pianoforte wire.	+79	−78	78.5	+12.9	−13 0	12.9	147.2	.039

TABLE XLIV.

MAGNETIC CHARACTERISTIC OF SOFT NICKELWIRE.

F	L	ρ	F	L	ρ
7.5	2.03	3.7	40	5.13	$\rho = 1.00 + .17\,F$
8	2.36	3.4	50	5.26	
9	2 73	3.3	60	5.36	
10	3.03	3.3	80	5.48	
12	3.58	3.35	100	5.56	
14	3.95	3.55	120	5.61	
16	4.21	3.8	140	5.65	
18	4.43	$\rho = 1.00 + .17\,F$	160	5.68	
20	4.55		180	5.70	
25	4.76		200	5.72	
30	4.92		[300	5.77	
35	5.04		500	5.81]	
Absolute Saturation				5.88	

TABLE XLV.

HYSTERESIS OF NICKEL.

			Ewing.			
	Soft Nickelwire.		Soft — Nickelwire. — Hard			
F	L_d	L_r	L_d	L_r	L_d	L_r
135	±5.64					
120	5.61					
110	5.59					
100	5.56					
90	5.53		$[F_1 = \pm 83]$			
80	5.49	5.48	±4.95		±4.15	
75			4.94	4.92	4.13	4.10
70	5.43	5.40	4.92	4.89	4.11	4.06
65			4.90	4.85	4.09	4.00
60	5.37	5.30	4.88	4.80	4.07	3.94
55	5.32	5.21	4.84	4.75	4.04	3.87
50	5.26	5 10	4.80	4.70	4.00	3.78
45	5.20	4.99	4.76	4.63	3.95	3.68
40	5.14	4.88	4 71	4.55	3.90	3.53
35	5.06	4.75	4.65	4.47	3.84	3.34
30	4.96	4.56	4.58	4.37	3.78	2 97
25	4.80	4.30	4.49	4.25	3.72	2.47
20	4.60	3.88	4.40	4.08	3.64	1.65
15	4.30	3.12	4.28	3.80	3.56	0
10	3.90	1.90	4.14	3.00	3.47	—1.70
5	3.33	—.40	3.95	—2.00	3.32	—2.63
0	±2.50		±3.56		±3.11	
$H =$	12.26		12.74		23.67	
$L =$	5.64		4.95		4.15	
$\eta =$	.01220		.01562		.038 49	

IX. NICKEL.

Some tests were made on commercial soft nickel wire.

The cross-section of the wire was = .0156 cm.^{2}.

The diameter, = .141 cm.

For the determination of the magnetic characteristic 45 wires, of 20 cm. length, were used, giving a joint cross-section of .7 cm.^{2}.

For the determination of the hysteresis 83 wires, of 1.23 cm.^{2} joint cross-section were used.

The wire was found magnetically softer than that of Ewing.

The magnetic characteristic is given in Table XLIV., one cycle of hysteresis in Table XLV., first column.

The denotations are the usual.

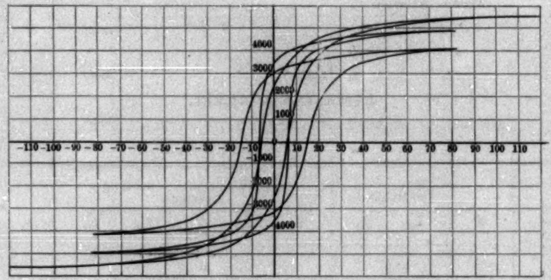


FIG. 14.—Nickel. Hysteretic Cycles.

As magnetic constants were found,

Coefficient of magnetic hardness.	Of magnetic saturation.	Of magnetic hysteresis.	Absolute saturation.
$\alpha = 1.00$	$\sigma = .17$	$\eta = .0122$	$L_\infty = 5.88$

for $F \geq 18$.

Hence,

$$\rho = 1.00 + .17\,F \qquad F \geq 18$$

$$H = .0122\left(\frac{L_1 - L_2}{2}\right)^{1.6}$$

The existence of the law of 1.6th power for the hysteresis of nickel has been proved by Kennelly, by two sets of tests communicated in the "*Electrical Engineer*," April 6th, 1892.

Ewing (page 87) gives two cycles, for soft and for hardened nickel wire. From these curves are taken the values given in Table XLV., second and third column.

The two cycles are not quite symmetrical, as given by Ewing. The figures given in Table XLV. are the mean values of the positive and of the negative part of the curve.

The results are,

		$\pm F$	$\pm L$	H	η	a	σ	L_∞
Soft nickel wire	Ewing	83	4.95	12.74	.0156			
Hardened " "		83	4.15	23.67	.0385			
Very soft " "		135	5.64	12.26	.0122	1.00	.17	5.88

These tests give for soft nickel about the same coefficient of hysteresis as for cast-iron, but a greater magnetic softness, while the value of absolute magnetic saturation, $L_\infty = \frac{1}{\sigma}$, is a little more than half that of cast-iron.

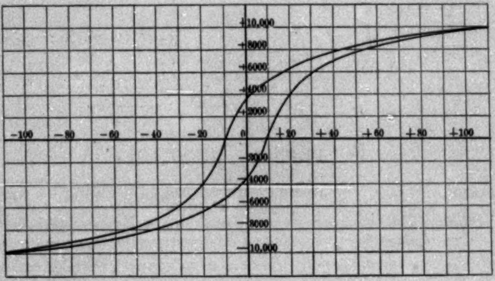


FIG. 15.—Cast Cobalt. Hysteretic Cycle. [Ewing.]

The magnetic characteristic is shown in Fig. 17, the three cycles of hysteresis in Fig. 14.

X. *Cobalt.*

Table XLVI. and Fig. 15 give an hysteretic cycle of cast-cobalt, from Ewing, page 89, which gives the results,

$\pm F$	$\pm L$	H	η
112	10.00	30.00	.0120

That means, cast-cobalt behaves magnetically very much like cast-iron, gives the same coefficient of hysteresis, and about the same value of magnetic saturation. Though it would be interesting to repeat these tests with different kinds of cobalt, of different degrees of softness.

TABLE XLVI.

HYSTERESIS OF CAST-COBALT (EWING).

F	L_d	L_r
112	±10.00	
100	9.8	9.75
90	9.6	9.45
80	9.4	9.1
70	9.1	8.7
60	8.8	8.3
50	8.35	7.75
40	7.8	6.95
30	7.2	5.8
20	6.4	4.0
10	5.2	.5
5	4.5	—2.0
0	±3.6	
$H =$	30.00	
$L =$	10.00	
$\eta =$	.01194 ~.012	

CHAPTER III.—RESULTS.

Combining now the results of the foregoing tests, we arrive at the conclusions:

1. *The dissipation of energy into heat by molecular hysteresis, during a complete cycle of magnetization, performed between the limiting values of magnetic induction L_1 and L_2, is expressed by the formula.*

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6},$$

where L_1 and L_2 very likely have to represent the metallic magnetic induction,

$$L = B - H = 4 \pi I,$$

while, when eddy—or Foucault—currents are induced by the cyclic variation of magnetization, the dissipation of energy is given by,

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6} + \epsilon N \left(\frac{B_1 - B_2}{2} \right)^2$$

where the first term is the loss by molecular hysteresis, the second term the loss by eddy-currents, N denotes the frequency.

2. *Beyond a certain minimum value of* M. M. F. *F_m, the metallic magnetic reluctivity, ρ (and consequently the inverse*

value of susceptibility, $\varkappa$, *which is,* $\frac{1}{\varkappa} = \frac{16 \pi^2}{10} \rho$) *follows the linear law,*

$$\rho = \sigma + a F$$

Below this minimum value of M. M. F. F_m *first the curve of alternating, then that of rising magnetism drops below, while the curve of decreasing magnetism rises above the curve derived from the linear law,* $\rho = a + \sigma F$.

3 *Beyond a certain minimum value* F_m, *that is for medium and high* M. M. F's. *all the main features of the magnetic properties of materials can be expressed by three constants,* a, σ, η,

a, *the coefficient of Magnetic Hardness,*
σ, " " " " *Saturation,*
η, " " " " *Hysteresis.*

Instead of a, σ *and* η *the three constants may be used,*

$L_\infty = \frac{1}{\sigma}$ *the value of absolute magnetic saturation.*

$F_0 = \frac{a}{\sigma}$ *that* M. M. F., *where half-saturation* $\frac{L_\infty}{2}$ *would be reached if the linear law of reluctivity holds already for* F_0.

$H_\infty = \eta L_\infty{}^{1.6}$ *the maximum value of hysteretic dissipation of energy, for absolute saturation.*

Then we have the equations:

Reluctivity,

$$\rho = a + \sigma F = \frac{F_0 + F}{L_\infty}$$

Hysteresis,

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6} = H_\infty \left(\frac{L_1 - L_2}{2 L_\infty} \right)^{1.6}$$

In the latter case the exponent 1.6 only covers an absolute number. while the coefficient of hysteresis H_∞ is of the dimension "work" or "energy," = (cm.^{2} g sec^{-2})

4. *These formulas hold for all kinds of wrought and cast-iron and steel, for nickel, and magnetite, and most likely for amalgam of iron,* hence apparently for all magnetizable materials.

For air simply σ and $\eta = O$, $a = 800$.

In Table XLVII. are given in the first six columns the three magnetic constants of all materials tested,

$$a, \sigma, \eta \text{ viz. } L_\infty, F_0, H_\infty.$$

TABLE XLVII.

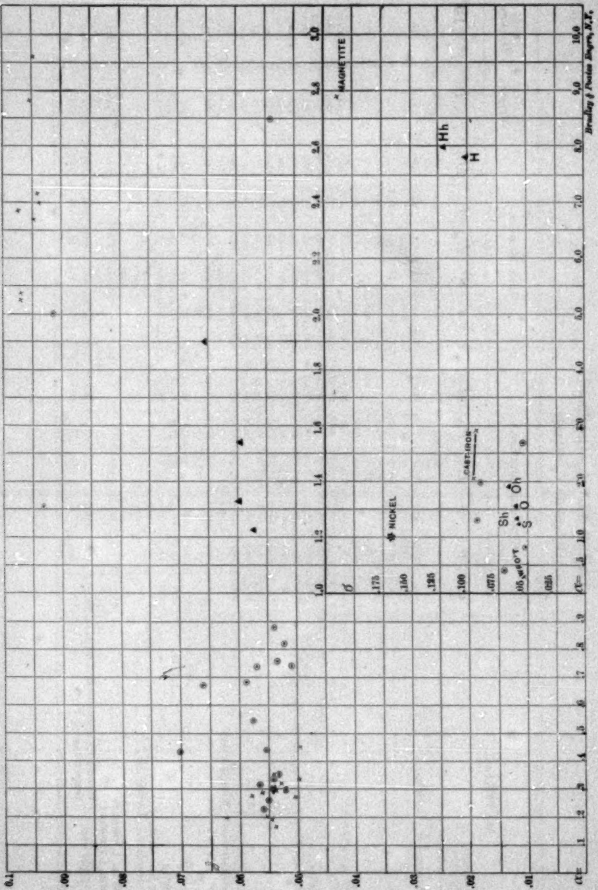
Magnetic Constants.

MATERIAL.	Centimetre Measure.						Inch Measure.						Centimetre Measure.	Inch Measure.
	a	σ	η	L_∞	F_o	H_∞	a^1	σ^1	η^1	L_∞^1	F_o^1	H_∞^1	$\gamma \times 10^6$	$\gamma^1 \times 10^6$
Wrought-Iron, Norway Iron	.166	.05435	.002275	18.40	3.05	15.17	.065	.0084	.00189	119	7.8	248	14.4	12.0
" Ordinary Wrought-Iron	.20	.05547	.003260	18.03	3.61	21.09	.079	.0086	.00271	116	9.2	345	20.6	17.1
Sheet-Iron ; thickness, δ = .042 cm.	.275	.058	.00350	17.24	4.74	21.01	.109	.0090	.00290	111	12.1	345	22.1	18.4
" " .026 "	.275	.0507		19.7	5.42		.109	.0079		127	13.8			
" " .038 "	.290	.0561		17.8	5.17		.114	.0087		115	13.1			
" " .071 "	.30	.0522		19.2	5.75		.118	.0082		122	14.6			
" " .071 "	.35	.0542		18.5	6.46		.138	.0084		119	16.4			
" " .015 " Ferrotype, Commercial.	.450	.04975	.00548	20.10	9.05	42.07	.177	.0077	.00455	130	23.0	690	34.6	28.7
" " .014 " " Annealed	.337	.04975	.00458	20.10	6.77	35.16	.133	.0077	.00380	130	17.2	574	28.9	24.0
" " .027 " Tin-Plate	.192	.05464	.002863	18.30	3.51	18.92	.076	.0085	.00237	118	8.9	310	18.1	15.0
" " .038 " "	.321	.05315	.004255	18.81	6.04	29.37	.127	.0082	.00353	122	15.4	480	26.8	22.3
Very Soft Iron-Wire (Ewing)	.20	.0635	.002	15.75	3.15	10.61	.079	.0096	.00166	104	8.0	174	12.6	10.5
Cast-Iron No. 1	2.40	.0940	.01300	10.66	25.5	36.18	.95	.0145	.0108	69	65	592	82.0	68.
" 2	2.43	.0943	.01317	10.60	25.8	36.26	.96	.0146	.0109	69	66	594	83.0	69
" 3	2.76	.0954	.01577	10.48	29.0	42.70	1.09	.0148	.0131	68	74	700	99.5	82
" 4	2.05	.09725	.01132	10.28	21.1	29.72	.81	.0151	.0094	66	54	486	71.4	59
" 5	2.34	.0950	.01267	10.52	24.7	34.56	.93	.0147	.0105	68	63	566	79.9	66
" 6	2.07	.0972	.01220	10.29	21.3	32.08	.82	.0150	.0102	67	54	526	77.0	64
" 7 (½ per cent. Aluminium)	2.37	.0976	.01365	10.25	24.3	35.77	.94	.0151	.0114	66	62	584	86.1	71
" 8 (½ " ")	2.92	.0948	.01459	10.55	30.8	39.94	1.15	.0147	.0121	68	78	655	92.1	76
Cast-Steel, Hardened	2.7	.054	.02792	18.5	50.0	187.7	1.07	.0084	.0232	119	127	3070	176.0	146
" Annealed	.88	.054	.00848	18.5	16.30	57.00	.347	.0084	.0070	119	41.4	935	53.5	44
"	2.00	.0913	.012	11.0	21.9	35.11	.79	.0141	.010	71	55.7	573	75.7	63
"	.82	.0521		19.2	15.74		.323	.0081		124	40.0			
"	.74	.0509		19.6	14.54		.292	.0079		127	37.0			
"	.76	.0534		18.7	14.23		.299	.0082		122	36.2			
"	1.26	.0931		10.7	13.64		.495	.0144		69	34.7			
" (Average of 5 Samples)	.736	.0568	.009	17.6	12.96	55.85	.290	.0088	.0075	114	33.0	912	56.8	47
"	.68	.0587		17.0	11.58		.268	.0061		110	29.5			
"	.545	.0575		17.4	9.48		.215	.0089		112	23.0			
"	.44	.0553		18.1	7.96		.173	.0086		116	20.2			

TABLE XLVII.—Continued.

Magnetic Constants.

MATERIAL.	Centimetre Measure.						Inch Measure.						Centimetre Measure.	Inch Measure.
	a	σ	η	L_∞	F_o	H_∞	a^1	σ^1	η^1	L_∞^1	F_o^1	H_∞^1	$\gamma \times 10^6$	$\gamma^1 \times 10^6$
Cast-Steel (Average of 5 Samples)	.35	.0535	.005	18.7	6.54	34.19	.138	.0083	.0041	121	16.6	560	31.6	26
"	.344	.0543		18.4	6.34		.136	.0084		119	16.1			
"	.430	.0700		14.3	6.14		.170	.0108		93	15.6			
"	.300	.0521		19.2	5.76		.118	.0070		127	14.6			
"	.300	.0543		18.4	5.52		.118	.0084		119	14.0			
"	.308	.0565		17.7	5.45		.121	.0087		115	13.8			
" No. 5	.260	.05511	.004573	18.15	4.72	29.82	.103	.0085	.00380	118	12.0	480	28.9	24.0
" No. 6	.232	.05577	.003181	17.95	4.16	20.38	.092	.0086	.00264	116	10.6	334	20.1	16.7
Mitis Metal	.300	.05444	.004281	18.37	5.51	28.46	.118	.0084	.00355	119	14.0	466	27.0	22.4
Galvanized Wire	.07	.066	.00349	15.15	10.15	17.03	.264	.0102	.00290	98	25.7	280	22.0	18.3
Welded Steel, Tool-Steel *Hh*	8.0	.121	.0748	8.28	66.1	138.9	3.15	.0187	.0620	54	168	2280	472	392
" " *H*	7.8	.105	.0613	9.53	74 3	142.6	3.08	.0162	.0508	62	188	2340	387	322
" " *Oh*	1.9	.066	.0267	15.16	28.8	130.5	.75	.0102	.0222	98	73	2140	168	140
" " *O*	1.54	.060	.0270	16.70	25.7	154.1	.61	.0093	.0224	108	65	2530	170	141
" " *Sh*	1.33	.060	.0190	16.70	22.2	108.4	.53	.0093	.0158	108	57	1780	120	100
" " *S*	1.22	.0575	.0145	17.40	21.2	88.3	.48	.0089	.0121	112	54	1440	91.5	76
Annealed Pianoforte Steel Wire (Ewing)			.0174						.0144					
Porous Iron	25.4	.604	.0425	1.66	42	6.03	10	.094	.0353	10.6	107	99	268	223
"	76	.71		1.41	107		30	.110		9.1	270			
Amalgam of Iron	500	1.12	.2314	.90	447	12.33	197	.173	.192	5.8	1140	203	1460	1220
Magnetite	8 9	.2132	.02348	4.69	41.7	17.56	3.5	.0330	.0195	30	10.6	287	148	123
Soft Nickel-Wire	1.00	.17	.01220	5 88	5.88	13.10	.394	.0263	.01015	38	14.9	215	77	64
Nickel-Wire, Soft (Ewing)			.0156						.0130					
" Hardened (Ewing)			.0385						.0320					
Cobalt, Cast			.01194						.0099					
Coiled Iron-Wire, Crossways	86.3	0	.0403				34		.0334				254	211
Laminated Iron, Crossways	31.6	0	00721				12.4		.0060				45.5	37.8
Iron Filings, 30 per cent. { Electro-Dynamometer Tests	62.5	.209	.0465	4.80	300	38.4	24.6	.0324	.0410	31.0	763	627	312	259
Iron Filings, 30 per cent. { Magnetometer Tests	77.5	.375	.0656	2.67	207	20.0	30.5	.0581	.0543	17.2	525	327	413	342



Bradley & Poates Engr's, N.Y.

FIG. 16.—Magnetic Constants.

In Fig. 16 are given the values of *a* as abscissæ with the corresponding values of *σ* as ordinates.

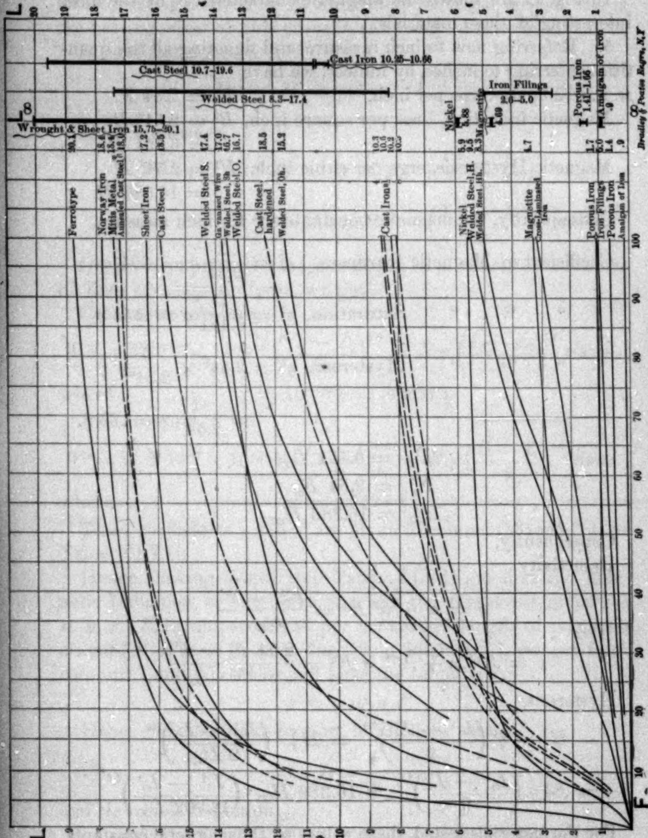


FIG. 17.—Magnetic Characteristics.

Wrought and sheet-iron, cast-iron and magnetite are marked by crosses × (since these two can not possibly be mistaken.)

Cast-steel is marked by circles ⊙, and welded steel by three-cornered dots, '

Mitis metal and Nickel are marked by six-cornered stars.

In Fig. 17 are shown the magnetic characteristics of the most interesting of these materials.

5. Referring now to inch measure, and denoting all the quantities referring to inches, by indices, we have

M. M. F., ampere-turns per inch, $F^1 = 2.54\,F$
Magnetic induction, lines per square inch, $B^1 = 2.54^2\,B = 6.451\,B$
Magnetic Hysteresis, ergs per cubic inch, $H^1 = 2.54^3\,H = 16.386\,H$

Consequently, the magnetic constants are for inch measure,

Coefficient of Magnetic Hardness, $\alpha^1 = \frac{1}{2.54}\,\alpha = .394\,\alpha$
" " " Saturation, $\sigma^1 = \frac{1}{6.451}\,\sigma = .155\,\sigma$
" " " Hysteresis, $\eta^1 = 2.54^3 \times \frac{1}{2.54^{3.2}}\,\eta = \frac{1}{2.54^{.2}}\,\eta = .83\,\eta.$

$$L^1_\infty = 6.451\,L_\infty$$
$$F^1_0 = 2.54\,F_0$$
$$H^1_\infty = 16.386\,H_\infty$$

Consequently,
Reluctivity,

$$\rho^1 = \alpha^1 + \sigma^1\,F^1 = \frac{F^1_0 + F^1}{L^1_\infty}$$
$$= .394\,\alpha + .155\,\sigma\,F^1 = \frac{2.54\,F_0 + F^1}{6.451\,L_\infty}$$

Hysteresis,

$$H^1 = \eta^1\left(\frac{L_1^1 - L_2^1}{2}\right)^{1.6} = H^1_\infty\left(\frac{L_1^1 - L_2^1}{2\,L^1_\infty}\right)^{1.6}$$
$$= .83\,\eta\left(\frac{L_1^1 - L_2^1}{2}\right)^{1.6} = 16.386\,H_\infty\left(\frac{L_1^1 - L_2^1}{12.902\,L_\infty}\right)^{1.6}$$

For the materials tested, these values of the magnetic constants in inch measure are given in column (7) to (12) of Table XLVII., as,

$$\alpha^1,\ \sigma^1,\ \eta^1,\ L^1_\infty,\ F^1_0,\ H^1_\infty.$$

6. From the Coefficient of Magnetic Hysteresis, the *loss of power* by molecular hysteresis in the iron under the influence of

an alternating current of N complete periods per second, that is the heating effect of this current, can easily be calculated. It is,

In centimetre measure,

$$W = N\, 10^{-7}\, H = \eta\, N\, 10^{-7} \left(\frac{L_1 - L_2}{2}\right)^{1.6} \text{ watts.}$$

In inch measure,

$$W^1 = N\, 10^{-7}\, H^1 = \eta^1\, N\, 10^{-7} \left(\frac{L_1^1 - L_2^1}{2}\right)^{1.6}$$

$$= .83\, \eta\, N\, 10^{-7} \left(\frac{L_1^1 - L_2^1}{2}\right)^{1.6} \text{ watts.}$$

Or, if we express the magnetization in kilolines, or thousands of lines of magnetic force, we get,

Centimetre measure,

$$W = \eta\, N 10^{-7} \times 1000^{1.6} \left(\frac{L_1 - L_2}{2}\right)^{1.6} = N\, \gamma \left(\frac{L_1 - L_2}{2}\right)^{1.6} \text{ watts.}$$

where $\gamma = 10^{-2.2}\, \eta = .00631\, \eta$

Inch measure,

$$W^1 = \eta^1\, N\, 10^{-7} \times 1000^{1.6} \left(\frac{L_1 - L_2}{2}\right)^{1.6} = N\, \gamma^1 \left(\frac{L_1 - L_2}{2}\right)^{1.6} \text{ watts.}$$

where $\gamma^1 = .00524\, \eta$

These coefficients γ and γ^1 are given in column (13) and (14) of Table XLVII.

Hence, making use of this Table XLVII., to find the Magnetic Induction, or Magnetization, and the Hysteresis, given the M. M. F. F, in ampere-turns per centimetre length of magnetic circuit [$F = .8\, H$ if H is the "field intensity"], we get from columns 1 and 2, α and σ and have the reluctivity,

$$\rho = \alpha + \sigma\, F$$

Hence the metallic induction, in kilolines per cm.^{2}.

$$L = \frac{F}{\rho}$$

and the whole induction,

$$B = L + H = L + .8\, F$$

Usually the H can be neglected, and $L = B$.

Taking now γ from the 13th column of Table XLVII., we get the dissipation of energy under the influence of an alternating current of N complete periods per second, in watts per cubic centimeter.

$$W = \gamma N L^{1.6}$$

where L is to be taken in kilolines.

To get B and W in inch measure, the M. M. F. F^1 being given in ampere turns per inch length of the magnetic circuit [consequently the field intensity $H = \frac{.8\ F^1}{2.54} = .245\ F^1$] we proceed in the same way, but take the values a^1, σ^1, γ^1 from columns 7, 8 and 14 of Table XLVII., and derive,

$$L^1 = \frac{F^1}{a^1 + \sigma^1 F^1}$$

$$W^1 = \gamma^1 N L^{1.6}$$

7. As M. M. F. here ampere-turns per unit length of the magnetic circuit are always used. To reduce to absolute measure, we have,

Field intensity, $H = \frac{4\pi}{10} F = \frac{4\pi}{25.4} F^1$

Susceptibility, $\varkappa = \frac{10}{16\pi^2 \rho}$

Permeability, $\mu = 4\pi\varkappa + 1 = \frac{10}{4\pi\rho} + 1$

Intensity of Magnetization, or

Magnetic Moment, $I = \varkappa H = \frac{L}{4\pi} = \frac{F}{4\pi\rho}$

Magnetic Induction, $$\begin{aligned} B &= L + H \\ &= 4\pi I + H \\ &= (4\pi\varkappa + 1) H = \mu H \\ &= \frac{F}{\rho} + \frac{4\pi}{10} F \end{aligned}$$

8. If now on the hand of the data collected in Table XLVII. and the curves represented in Fig. 17, we look over the numerical values of the magnetic constants of different materials, we see, that in

Wrought-Iron and Sheet-Iron.

The Coefficient of

Magnetic Hardness, a,	varies from	.166	to	.450
Magnetic Saturation, σ,	"	.04975	"	.058
Magnetic Hysteresis, η,	"	.002275	"	.00548
Consequently the value of absolute saturation, L_∞,	"	17.24	"	20.10

The variations are considerable enough to make it advisable everywhere, where a somewhat greater accuracy of calculation is required, especially to determine the individual constants of the material employed, which can be done easily, since only three observations are required hereto, two of L, or ρ, and one of H.

As a fair average of good wrought or sheet-iron we can consider an iron of the constants,

$$a = .30 \qquad \sigma = .055 \qquad \eta = .0030$$
$$L_\infty = 18.0$$

In Tables XLVIII., XLIX., L. and Figs. 18, 19, 20 the magnetic curves of this average wrought-iron are given.

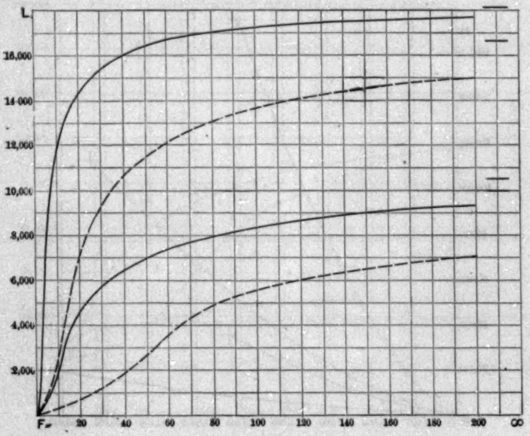


FIG. 18.—Average Materials, Magnetic Characteristics.

Cast-Iron.

Although cast-iron, as the raw-material, should be expected to vary considerably, nevertheless the difference between the eight samples tested—though derived from different sources—are remarkably small, the

Coefficient of				
Magnetic Hardness, a,	varying from	2.05	to	2.92
Magnetic Saturation, σ,	"	.0940	"	.0976
Magnetic Hysteresis, η,	"	.0113	"	.0158
Consequently the value				
of absolute saturation, L_∞,	"	10.25	"	10.66

Hence of *cast-iron* it is much oftener permissible to take an average set of magnetic constants,

$$a = 2.40 \qquad \sigma = .095 \qquad \eta = .013$$
$$L_\infty = 10.5$$

In Tables XLVIII., XLIX., L. and Figs. 18, 19, 20, the magnetic curves of this cast-iron are given.

Welded Steel.

That is, that kind of steel which can be hardened, evidently varies in its constants enormously with its degree of hardness.

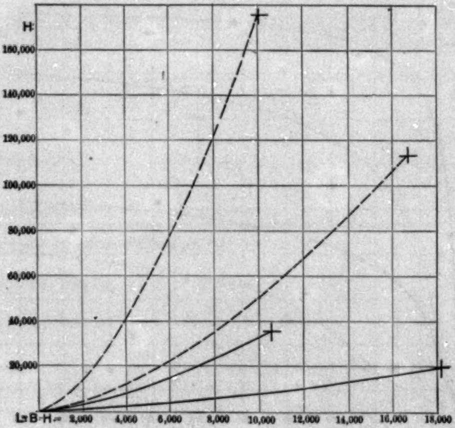


FIG. 19.—Average Materials. Curves of Hysteresis.

For instance the tests referring to one and the same material of different degrees of hardness, give the variations in

Magnetic Hardness, a,	from	1.22	to	8.0
Magnetic Saturation, σ,	“	.0575	“	.111
Magnetic Hysteresis, η,	“	.0145	“	.0748
Absolute Saturation, L_∞	“	8.28	“	17.40

In comparison with cast material the relatively high coefficient of hysteresis is remarkable, as even for the softest annealed condition it is higher than the average of cast-iron.

Tables XLVIII., XLIX., L. and Figs 18, 19, 20, give two sets of curves, in dotted lines, of soft material,

$$\alpha = 1.33 \qquad \sigma = .060 \qquad \eta = .020$$

$$L_\infty = 16.67$$

and glass-hard material,

$$\alpha = 8.0 \qquad \sigma = .10 \qquad \eta = .070$$

$$L_\infty = 10.00$$

Coming now to

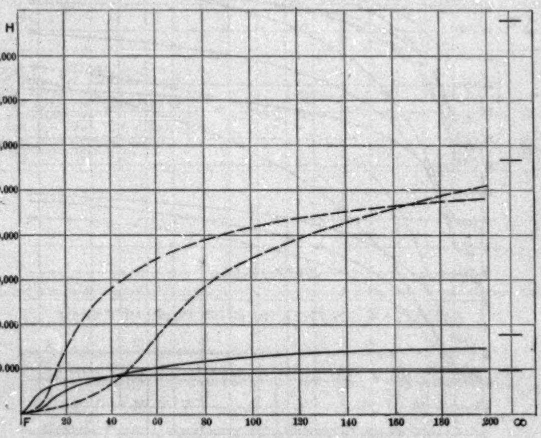


Fig. 20.—Average Materials. Curves of Hysteresis.

Cast-Steel.

We see that no averaging is possible at all, but cast-steel comprises and includes the whole range of materials, giving a continuous and unbroken range from the softest kind of sheet-iron down to and beyond cast-iron and to medium hard welded steel, as a glance on Tables XLVII., LI. shows and especially on Fig. 16 (where the cast-steel is marked by circles), and Fig. 21, where some cast-steel characteristics are shown as drawn lines together with the Norway-iron curve (*N*), the average wrought iron curve (*W*), the soft welded steel curve (*s*) and the cast-iron curve (*C*) as dotted lines.

Magnetic Hardness, α,	from	.232	to	2.7
Magnetic Saturation, σ,	"	.0509	"	.0931
Magnetic Hysteresis, η,	"	.00318	"	.0279
Absolute Saturation, L_∞	"	10.7	"	19.6

Consequently, for good annealed cast-steel] of high permeability—as it can be got now very easily—the average wrought-iron curves can be used, since they represent also a fair average of soft annealed cast-steel and of mitis metal.

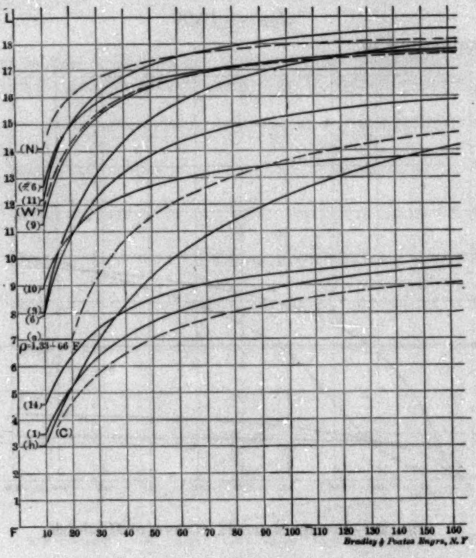


FIG. 21.—Cast-Steel. Magnetic Characteristics.

Poorly annealed cast-steel of high permeability will give a curve similar to that of soft welded-steel, and cast-steel of low permeability is as good as identical with cast-iron, as will be best seen on Fig. 21.

In Table XLVIII. are given magnetic constants of average materials, in Tables XLIX. and L. the magnetic characteristics and curves of hysteresis calculated therefrom. In Figs. 18, 19, 20, these curves are shown, the two welded-steel curves dotted, the cast-iron and wrought-iron curves drawn.

TABLE XLVIII.

MAGNETIC CONSTANTS OF AVERAGE MATERIALS.

MATERIAL.	Coefficient of Magnetic Hardness α	Coefficient of Magnetic Saturation σ	Coefficient of Magnetic Hysteresis η	Absolute Saturation L_∞	$F \geqq$
Average *Wrought* and *Sheet-Iron*, *Soft Annealed Cast-Steel* and *Mitis Metal*	.3	.055	.003	18.2	7
Average *Cast-Iron*, *Cast-Steel* of *Low Permeability*	2.4	.095	.013	10.5	18
Average *Soft Steel*, *Hard Cast-Steel* of *High Permeability*	1.33	.06	.02	16.7	40
Average *Glass-Hard Steel*........	8	.1	.07	10.0	90

TABLE XLIX.

MAGNETIC PROPERTIES OF AVERAGE MATERIALS.

	Average Wrought and Sheet-Iron.		Average Cast-Iron.		Average Soft Steel.		Average Glass-Hard Steel.	
L	*F*	*H*	*F*	*H*	*F*	*H*	*F*	*H*
1	2	.2	7	.8	6	1.3	26	4.4
2	2	.6	10	2.5	9	3.8	42	13.4
3	3	1.1	13	4.8	11	7.3	54	25.6
4	3	1.7	16	7.5	13	11.6	66	40.6
5	4	2.5	23	10.8	15	16.6	83	58.0
6	4	3.3	33	14.4	17	22.2	120	77.8
7	5	4.3	56	18.5	20	28.4	188	99.4
8	5	5.3	81	22.9	23	35.2	320	123.1
9	6	6.4	148	27.6	28	42.4	720	148.5
10	7	7.5	500	32.6	35	50.2	∞	175.8
11	8	8.8	∞	35.5	44	58.5	[$L_\infty = 10.0$]	
12	11	10.1	[$L_\infty = 10.5$]		58	67.3		
13	14	11.5			79	76.4		
14	18	12.9			117	86.1		
15	26	14.4			200	96.1		
16	30	16.0			500	106.6		
17	67	17.6			∞	113.7		
18	600	19.3			[$L_\infty = 16.7$]			
	∞	19.6						
	[$L_\infty = 18.2$]							

TABLE L.

MAGNETIC PROPERTIES OF AVERAGE MATERIALS.

	Average Wrought and Sheet-Iron.		Average Cast-Iron,		Average Soft Steel.		Average Glass-HardSteel	
F	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*
1	.4	.1						
2	1.7	.4			.2			
3	3.8	1.6						
4	5.6	3.0			.6			
5	7.5	4.8	.7	1	.8	1	.1	
6	9.9	6.4			1.0			
7	10.1	7.6						
8	10.8	8.5			1.7			
9	11.4	9.3						
10	11.8	9.0	1.9	2	2.5	5	.3	1
12	12.5	10.8			3.5			
15	13.4	12.1	3.7	7	5.0	17	.5	2
20	14.3	13.5	4.6	10	7.0	29	.7	4
25	14.9		5.2	12	8.3	38	.9	5
30	15.4	15.1	5.7	14	9.3	44	1.2	7
35	15.7		6.1	15	10.1	50	1.5	9
40	16.0	16.0	6.5	17	10.7	55	1.9	13
45	16.2		6.8	18	11.2	59	2.3	17
50	16.4	17.0	7.0	19	11.5	63	2.7	22
60	16.7		7.4	20	12.1	69	3.5	33
70	16.9		7.7	22	12.6	73	4.3	46
80	17.0		8.0	23	13.0	77	4.9	55
90	17.1		8.2	24	13.4	80	5.2	63
100	17.3	18.0	8.4	25	13.6	83	5.5	69
120	17.4		8.7	26	14.1	87	6.0	78
140	17.5		8.9	27	14.4	90	6.4	85
160	17.6		9.1	28	14.6	93	6.7	91
180	17.6		9.2	29	14.8	95	6.9	97
200	17.7	19.0	9.3	29	15.0	96	7.1	102
Absolute Saturation	18.2	19.6	10.5	36	16.7	114	10.0	176

TABLE LI.

MAGNETIC CHARACTERISTICS OF CAST-STEEL.

(1)			(2)			(3)			(4)			(5)			(6)			(7)		
F	ρ obs.	ρ calc.	*F*	ρ obs.	ρ calc.	*F*	ρ obs.	ρ calc.	*F*	ρ obs.	ρ calc.	*F*	ρ obs.	ρ calc.	*F*	ρ obs.	ρ calc.	*F*	ρ obs.	ρ calc.
11	3.00	3.00	17.5	1.71	1.73	12	1.32	1.35	12	1.41	1.40	8	1.72		21	1.89	1.91	10	1.51	
27	4.47	4.47	25	2.15	2.13	18	1.66	1.66	14.5	1.56	1.54	10	1.70		28.5	2.36	2.35	18	1.62	1.58
76	9.00	8.94	32.5	2.52	2.51	25	2.02	2.01	25	2.08	2.09	15	1.76		37.5	2.88	2.88	34	2.46	2.48
92	10.34	10.40	63	4.10	4.10	34	2.46	2.47	41	2.95	2.95	20	1.95		45	3.32	3.32	76	4.92	4.92
			73	4.61	4.62	61	2.86	2.85	79	4.98	4.98	25	2.17		50	3.63	3.62	92	5.83	5.83
			85	5.28	5.26	69	4.26	4.25	97	5.95	5.94	Average of 5 Samples.			62	4.32	4.33			
$\alpha =$		2.00			.82			.74			.76			.736			.68			.545
$\sigma =$		.0913			.0521			.0509			.0534			.0568			.0587			.0575
$\eta =$		.012												.009						
$L_\infty =$		11.0			19.2			19.6			18.7			17.6			17.0			17.4
$\frac{\alpha}{\sigma} = F_\circ =$		21.9			15.74			14.54			14.23			12.96			11.58			9.48

TABLE LI.—Continued.

MAGNETIC CHARACTERISTICS OF CAST-STEEL.

	(8)			(9)			(10)			(11)			(12)			(13)			(14)			(15)
	F	ρ obs.	ρ calc.	F	ρ obs	ρ calc.	F	ρ obs.	ρ calc.	F	ρ obs.	ρ calc.	F	ρ obs.	ρ calc.	F	ρ obs.	ρ calc.	F	ρ obs.	ρ calc.	Average of 5 Samples.
	11	1.04	1.05	12	.99	.99	8	1.04		15	1.08	1.08	10	.84	.84	10	.85	.87	44	5.34	5.35	
	16	1.33	1.33	13.5	1.08	1.08	13	1.31	1.32	22	1.45	1.45	19	1.34	1.33	12	.97	.98	61	6.95	6.94	
	21	1.60	1.60	21	1.48	1.48	34	2.83	2 81	34	2.16	2.17	32	2.04	2.04	15	1.14	1.15	78	8.52	8.52	
	28	1.99	1.99	24.5	1.67	1.67	76	5.73	5.75	54	3.12	3.11	76	4.43	4.43	19	1.42	1.38	95	10.10	10.10	
	40	2.64	2.65	30	1.98	1.97	92	6.90	6.87	70	3.95	3.95	95	5.45	5.46	23	1.62	1.62				
	51	3.22	3.26	35	2.24	2.25				95	5.25	5.25				36	2.36	2.34				
	76	4.67	4.64	41	2.60	2.57										46	2.88	2.91				
	95	5.71	5.69	56	3.39	3.39										61	3.79	3.76				
				65	3.87	3.87										73.5	4.45	4.46				
				73	4.31	4.31										94	5.63	5.62				
$\alpha =$			.44			.344			.43			.300			.300			.308			1.26	.35
$\sigma =$			.0553			.0543			.070			.0521			.0543			.0565			.0931	.0535
$\eta =$																						.005
$L_\infty =$			18.1			18.4			14.3			19.2			18.4			17.7			10.7	18.7
$\frac{\alpha}{\sigma} = F_0 =$			7.96			6.34			6.14			5.76			5.52			5.45			13.64	6.54

With regard to cast-iron, I must remark, however, that some tests of Ewing and others show magnetizations as high as $L = 16,000$, while I was never able to reach much beyond $L = 10,000$.

It must be assumed, therefore, that either the linear law of magnetic reluctivity, $\rho = \alpha + \sigma F$ ceases to hold for higher magnetizations than I was able to reach—which is not likely, however,—or we must assume that there exist kinds of cast-iron far superior to all the samples I ever came across, and if so, then very great improvements are possible in the manufacture of cast-iron for magnetic purposes.

CHAPTER IV.—HETEROGENEOUS MATERIALS.

I. COILED WIRE.

Since armatures of dynamo electric machines have quite extensively been wound of iron wire, I thought it interesting to determine the magnetic reluctance of wire against a magnetic flux passing crosswire through it.

Therefore I wound on a brass wire of $\frac{1}{8}$ in. diameter 6 layers of the galvanized wire, tested in Chapter II., IV., 8, the adjacent turns closely touching each other (with only the thin film of zinc between, which the wire is covered with). The consequent layers were wound always in the same direction into the interstices between the turns of the layer underneath, starting each layer separately. The outside diameter was $\frac{7}{8}$ in., so that the spiral just fitted into the holes in the pole faces of the magnetometer, which have a cross-section of 4 cm^2. The projection of the 6 layers of wire upon a plane vertical to the axis was very nearly 3.9 cm^2. The magnetism passed in the direction of the axis of the spirals, thereby crossing from turn to turn. The magnetic induction L and the magnetic reluctivity ρ were calculated with regard to the whole space taken up by the spirals, 4 cm.^{2}, no allowance being made for the hole in the middle, since it only amounted to 2 per cent. of the cross-section.

The magnetic reluctivity of this heterogeneous body was found remarkably high, about one-ninth that of common air; no decided trace of saturation was perceptible, which indeed is not astonishing, since the highest value of induction reached in the tests was only 1,900 lines per cm^2.

The magnetic characteristic is given in Table LII.

The metallic magnetic reluctivity was found $\rho = 86.3$.

The different readings indeed varied considerable, an average of 4 per cent., but these variations were entirely irregular and to be expected, since the magnetic reluctivity was very small, and the smallest fractional standard to balance with is $\frac{1}{40}$ cm.^{2} sheet-iron, of which quarters can be estimated, so that, when taking the average of two readings, a sensitivity of about 10 to 15 lines of magnetic force per cm.^{2} can be reached by the instrument.

Two magnetic cycles of this coiled wire are given in Table LIII.

Their results and the constants of the magnetic characteristics are,

$\pm F$	$\pm L$	H	η	a	σ
35	.37	.505	.0393		
140	1.61	5.60	.0414		
		av. $\eta =$	.0403	86.3	~ 0
			$\sim .04$		

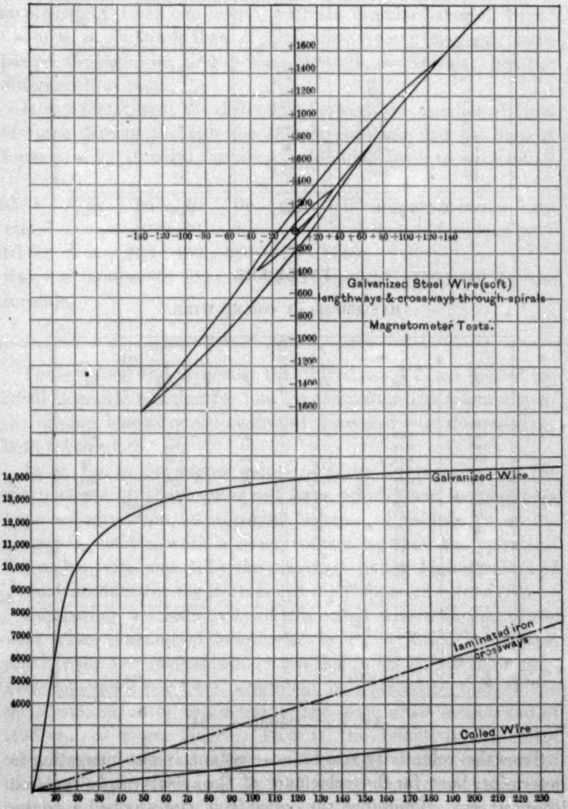


FIG. 22.—Coiled wire and cross laminated iron.

Fig. 22 gives the magnetic characteristic of this coiled wire, and of the wire magnetized lengthwise, and the two cycles of hysteresis.

TABLE LII.

MAGNETIC CHARACTERISTIC OF COILED WIRE.

F	L	ρ	$\triangle \rho$	= %
8.5	104	82.0	+4.3	+5
11.5	144	80.0	+6.3	+7
26	297	87 5	—1.2	—1
31	364	85.0	+1.3	+1
52	575	90.5	—4 2	—5
66	730	90.4	—4.1	—4
100	1228	81.5	+4.8	+5
116	1395	83.0	+3.3	+4
140	1555	90.0	—3.7	—4
155	1670	92.8	—6.5	—8
Av. ρ =		86.3	±4.0	±4.4

TABLE LIII.

HYSTERESIS OF COILED WIRE.

	(1)			(2)	
F	L_d	L_r	F	L_d	L_r
±140	±1.61		35	±.370	
130	1.52	1.50	30	.335	.315
120	1.43	1.38	25	.295	.255
110	1.35	1.27	20	.255	.195
100	1.25	1.15	15	.215	.130
90	1.17	1.04	10	.170	.060
80	1.08	.91	5	.115	—.010
70	.99	.77	0	±.065	
60	.88	.64			
50	.77	.50			
40	.66	.36			
30	.55	.22			
20	.43	.08			
10	.30	—.06			
0	±.18				
H =	5.60			.505	
L =	1.61			.37	
η =	.0414			.0393	

$$\text{Av. } \eta = .04035 \sim .04.$$

Since the reluctivity was found constant, it was interesting to determine, how far the reluctance of the spirals can be replaced by an air gap. Therefore the coiled iron wire was laid into the holes in the pole-faces at the one side of the magnetometer, and in the holes in the pole-faces at the other side of the instrument two Norway iron cylinders, of 4 cm.^{2} cross-section and 8 cm. length, were laid, with plane faces against each other, and their distance adjusted until equilibrium was restored. The distance from pole face to pole face was 10.9 cm., and it was found, that for M. M. F. of $F > 80$ the spirals can be perfectly balanced by an air gap of 1.852 cm. length, between circular faces of 4 cm.^{2}

For M. M. F's. lower than $F \leqq 80$ more lines of magnetic force passed through the air-gap than through the spirals; but the difference was small.

It was found, that the difference between the number of lines of force passing through the spirals, per cm.^{2}, and the lines of force passing through the air gap (divided by 4, to reduce to 1 cm.^{2})

at $F =$	20	40	60	80	100	ampere turns per cm.
was $\delta L =$	40	30	20	10	0	lines of force per cm.^{2}
while $L =$	230	460	680	910	1140	

was the number of lines of force per cm.^{2}, calculated by the formula,

$$L = \frac{F}{86.3 \times 10^{-3}}$$

These values, and especially the differences δL, are indeed too small to decide whether for low magnetization the reluctivity of the air-gap has increased or that of the coiled wire decreased, or both taken place.

In so far as for higher values of L the Norway iron at the sharp edges of the circular end faces, which form the gap, may approach saturation, an apparent increase of reluctivity of the air gap is possible, while a closer contact between the spirals of the coiled wire, caused by the magnetic pull at higher values of F, may account for the decrease of their apparent reluctivity.

Comparing the reluctivity of this coiled wire with that of the wire when magnetized lengthwise, in Table XXXIV. we see, that for the low magnetizations reached in the spirals their magnetic reluctance per 1 cm. length can be replaced by that of the same iron, including an air gap of the same cross section and of .106 cm., $\sim \frac{1}{9}$ cm. length. That is, the reluctivity of coiled wire is equal to that of solid iron including about one-ninth of its length air reluctance. Indeed, these numerical values are conclusive only for the conditions of this particular test, and will differ, when different sizes of wire are used, when the wire is wound on under strain, to make a closer contact, or when insulated wire is used, and thereby adjacent turns are separated further, and will differ with the magnetization reached.

But what these tests prove is, that the magnetic reluctivity of coiled wire against a magnetic flux passing crosswise through the wire, is enormously higher than that of solid iron, is under circumstances equivalent to one-ninth of its length in air resistance.

As before stated, the reluctivity of the coiled wire is equivalent to that of solid iron including 10.6 per cent. of its length in air reluctance. The distance between the pole faces of the magnetometer being 10.9 cm., the spirals were equivalent to solid iron plus an air reluctance of 10.9 × .106 = 1.15 cm. length and 4 cm.^{2} cross section. But they were directly balanced by an air gap of 1.852 cm. length between circular faces of 4 cm^2. Hence, to calculate the reluctance of air gaps by the reluctance of air of the length of the air gap and the cross section of its faces,

$$\rho = \frac{\text{length}}{\text{crosswise}}$$

as is even done in the new edition of Silvanus Thompson's "Dynamo Electric Machinery," introduces a very serious error when the length of the gap is considerable compared with its cross section, caused by the spreading out of the lines of magnetic force. For instance, in the case mentioned here, the cross section of the faces being circular and 4 cm.^{2}, the length of the gap 1.852 cm., the usual manner of calculation, without taking into consideration the spreading out of the lines, will bring out the reluctance 61 per cent. too large. The reluctance of this air gap of $l = 1.85$ cm. between circular pole faces of 4 cm.^{2} = 2.26 cm. diameter, is equal to the reluctance of an air cylinder of $l = 1.85$ cm. and 6.44 cm.^{2} cross section, that is 2.86 cm. diameter, or the diameter has to be increased approximately by $\frac{l}{3}$, one-third the length of the gap. Hence,

The reluctance of an airgap of the length l between cylindrical pole faces of the diameter d is approximately equal to the reluctance of an air cylinder of the same length l but of the diameter $d + \frac{l}{3}$, hence it is,

$$\rho^0 = \frac{l}{(d + \frac{l}{3})^2 \frac{\pi}{4}}$$

or, if the same is true for rectangular air gaps, as will be in rough approximation, if a and b are the sides of the rectangle, the reluctance is:

$$\rho^{\circ} = \frac{l}{(a + \frac{l}{3})(b + \frac{l}{3})}$$

as long indeed only as the length l of the gap is not greater than its diameter.

I have dwelled upon this point somewhat longer, not that I consider the results as conclusive, but because I consider it as a good topic for further investigation.

One more point is remarkable with these wire spirals:

The cofficient of hysteresis is for cross magnetization:

$$\eta = .04$$

more than ten times larger than for length magnetization:

$$\eta = .0035.$$

This is astonishing, the more, as under cross magnetization the conditions resemble those of an open magnetic circuit.

In my former paper I have already pointed out that in an open magnetic circuit the coefficient of hysteresis must be apparently larger than in a closed circuit, since in the closed circuit the magnetization is more homogenous than in an open circuit where the density decreases near the air gaps.

Since the average of the 1.6th powers of different quantities is larger than the 1.6th power of the average of the different quantities, the coefficient of hysteresis, if the magnetization is not homogenous, must come out larger by the ratio of $\frac{\text{average of 1.6th power}}{\text{1.6th power of the average}}$ of different magnetic densities. In my former paper I proved this on the instance of a magnetic circuit with two air gaps.

Here in the case of the coiled wire the magnetization must be enormously heterogenous. While the greatest part of the iron is magnetized very low, at those linear places where the turns touch each other, high saturation may be already reached. Besides, obviously a large amount of magnetism does not cross from turn to turn, but passes along the wire in spirals from pole to pole, so that really the iron is magnetized much higher than the readings give, which represent only the axial component of the magnetism. For, at the M. M. F. $F = 100$, between adjacent wire turns, is a difference of magnetic potential: $F \times d$, where d is the diameter of the wire; that is: 15.7 ampere-turns.

Now the average length of a turn is 4 cm., and therefore act spirally upon the wire $F = 4$ ampere-turns per cm., giving an induction $L = 2000$, of which only an imperceptibly small portion counts in axial direction. That is, in other words, the axis of maximum magnetization in the iron does not coincide with the direction of M. M. F. in which the readings are taken but a circular magnetization is superposed upon the length magnetization.

Furthermore, it is not impossible that in such a heterogenous body as drawn wire the magnetic constants are different axially and radially. But a still better explanation of the high coefficient of hysteresis of these spirals will be pointed out in the next chapter.

II. LAMINATED IRON.

The test pieces of thick tin plate of $\delta = .0378$ cm. thickness described in Chapter II., IV. *f*, Table XXXIV. were cut into pieces of 1 in. × $\frac{3}{4}$ in., built into a pile, clamped together and soldered, forming a solid block of iron with intervening layers of tin, that is: laminated crosswise; or in the direction perpendicular to the direction of the M. M. F., of 16 cm. in length and 2.53 cm. × 1.90 cm. = 4.8 cm.^{2} cross section.

The block contained 26 sheets per cm., and consequently 26 gaps filled with tin per cm. length. Each gap was equivalent to an airgap of about $\frac{1}{100}$ cm., as will be seen hereafter.

TABLE LIV.

MAGNETIC CHARACTERISTIC OF LAMINATED IRON, ACROSS THE LAMINATION.

F	L	ρ	$\Delta\ \rho$	= %
7	.22	31.5	+ .1	+ .3
11	.33	32.3	− .7	−2.2
16	.50	32.0	− .4	−1.2
29	.97	30.0	+1.6	+5.3
39	1,24	31.5	+ .1	+ .3
50	1,63	30.7	+ .9	+2.8
53	1,62	32.7	−1.1	−3.5
65	2,09	31.2	+ .4	+1.2
66	2,04	32.3	− .7	−2.2
82	2,56	32.0	− .4	−1.2
102	3.29	31.0	+ .6	+1.9
120	3.82	31.2	+ .4	+1.2
165	5.12	32.2	− .6	−1.9
	Av...	31.6	± .6	±2

TABLE LV.

HYSTERESIS AND MAGNETIC CONSTANTS OF LAMINATED IRON, ACROSS THE LAMINATION.

	F	L	H	η	a	σ
Laminated with 26 plates per cm., each gap about $\frac{1}{100}$ cm.	70	2.20	1.63	.00732		
	40	1.26	.65	.00712		
	Average			.00722	31.6	~0
Material proper				.00426	.321	.05315

Magnetometer tests gave for the reluctivity the values given in Table LIV. The magnetic characteristic is shown as dotted line in Fig. 22. As seen, up to the highest magnetization reached, of $L = 5.12$, the reluctivity is constant, $\rho = 31.6$, and the differences between the observed values and the average value are entirely irregular, and not larger than the errors of observation account for, which in such a case are necessarily larger than with homogenous materials of high permeability. The results of two magnetic cycles of this cross-laminated iron are given in Table LV, showing a coefficient of hysteresis $\eta = .00722$, while the material proper had the coefficient of hysteresis $\eta = .00426$, that is somewhat more than half the former value.

Since the magnetic reluctivity of the material proper is known, from the observed reluctivity of the laminated block and the number of sheets per cm. = 26, we can compute the approximate width of air space equivalent to each layer of tin or gap between adjacent plates and find it equal to about $\frac{1}{100}$ cm. Probably the gap is less in reality. In the average, the reluctivity of laminated sheet-iron with the laminæ very close together as in this case, is about 30 times higher than that of the sheet-iron in the direction of lamination. But even across the lamination, laminated sheet-iron is still superior to coiled wire. The coefficient of hysteresis across the lamination is still about 70 per cent. higher than along the lamination, .0722 against .0426, though not by far as much higher as in the case of the coiled wire.

This higher value of hysteresis may be partly due to a higher coefficient of hysteresis perpendicular to rather than in the plane of the sheet-iron. But mainly I believe it is caused by the unequal magnetic density at the different points of the cross-section. The separate laminæ are evidently not absolute planes, and consequently the interstices between them not of a constant width, but the plates at some places almost in molecular contact, at other points farther apart. That means that each gap between adjacent laminæ is not of constant width, but of a width varying from almost nothing to say .01 cm. But, since the reluctance of each gap is about 30 times that of each lamina, the greatest part of the M. M. F. is consumed in the gap, and the magnetic lines of force will crowd together at those points where the adjacent laminæ come nearest together. In the iron consequently the magnetism will not flow perpendicularly across, but will largely spread sideways from the point nearest to the preceding lamina to the point nearest to the next lamina, and in consequence of this irregular cross-magnetization the magnetic density in the iron must be larger than the magnetic density in the direction of the M. M. F., and consequently η comes out larger. Numerical figuring shows that this fact fully accounts for the higher value of η without any further assumption. This effect must become less when the gaps between the laminæ are larger, for instance, sheets of paper are placed therein. Though I must leave this question also for future research.

III. Iron Filings.

Remarkable results were obtained by testing the magnetic behavior of iron filings. The iron filings were produced by clamping a large number of sheets of the iron tested in Chapter I. together, and cutting notches therein by means of a rotary cutter of $\frac{5}{16}$ in. = .79 cm. width, thereby producing fine needle-like iron chips. Tests were made by the electro-dynamometer method and by the magnetometer method. In the dynamometer method the same magnetizing spools were used as in Chapter I., and by means of these spools and two U-shaped end-pieces a box-like receptacle formed. This was filled with the iron filings, and by vigorously beating it against the table the filings were made to settle down.

In the magnetometer method a brass tube of 4 cm.^{2} cross-section and 8 cm. length was filled with these iron filings, which were enclosed between two cylindrical Norway iron pieces, and there-after tested. The magnetic constants were found very much higher than in the electro-dynamometer tests.

Since in the electro-dynamometer tests the iron filings by beating to make them settle closer together had evidently assumed a kind of horizontal stratification, that is, stratification in the direction of the magnetic flux, while in the magnetometer tests the tube containing the filings had been filled from the end, and consequently the filings had assumed a stratification perpendicular to the direction of the magnetic flux, a higher magnetic hardness was to be expected.

Therefore a larger tube of 17.8 cm.^{2} cross section was secured, a slot cut in the tube lengthwise, the tube fastened between the cylindrical pole blocks, and then filled with iron filings from the top through the slot, and by vigorously beating the filings were made to settle down in a stratification in the direction of the magnetic flux, the same as in the electro-dynamometer tests. In all these tests approximately 30 per cent. of the volume filled by the filings consisted of iron.

One more test was made by wetting the iron filings with turpentine and stamping them tight into the brass tube of 4 cm.^{2} cross section.

1. *Electro-dynamometer Tests.*

Length of magnetic circuit, 30 cm.
Cross-section " " 13.7 cm.^{2}

Tests were made with the frequencies of 180 and 114 complete periods per second, and a few readings with still lower frequency.

The results are given in Table LVI., in the usual denotation.

TABLE LVI.

ELECTRO-DYNAMOMETER TESTS OF IRON FILINGS.

	F	B	L	H obs.	ρ	μ	η_B	H calc.	$\triangle$	= %	η_L	H calc.	$\triangle$	= %
	(1) 180 Complete Periods per Second, $N \doteq 180$.													
	24	323	293	400	82.0	10.8	[.0387]	470	+ 70	[+15]	[.0455]	470	+ 70	[+15]
	27.7	420	385	700	72.0	12.2	.0445	720	+ 20	+3	.0511	730	+ 30	+4
	36.6	523	477	1000	77.0	11.4	.0447	1010	+ 10	+1	.0518	1030	+ 30	+3
	41.3	597	545	1220	76.0	11.6	.0441	1260	+ 40	+3	.0511	1270	+ 50	+4
	51	738	674	1900	$\rho = 64 + .218\ F$	11.5	.0489	1780	−120	−7	.0566	1790	−110	−6
		750	690	1750			.0439	1820	+ 70	+4	.0502	1860	+110	+6
		826	740	2200			.0473	2120	− 80	−4	.0557	2100	−100	−5
	70	980	892	2750		11.2	.0450	2790	+ 40	+2	.0523	2800	+ 50	+2
	85	1130	1024	3480		10.7	.0454	3500	+ 20	+1	.0531	3490	+ 10	+0
	98	1270	1147	4280		10.4	.0463	4230	− 50	−1	.0557	4190	− 90	−2
	112	1410	1270	5120		10.1	.0468	4990	−130	−3	.0554	4930	−190	−4
	Absolute Saturation..... } $L_\infty = 4590$.					Av. η =	.0457		± 60	±2.9	.0533		± 80	±3.3
	(2) 114 Complete Periods per Second, $N = 114$													
	49.4	580	531	1070	74.0	11.8	.0405	1050	− 20	−2	.0467	1050	− 20	−2
	47	724	665	1420	$\rho = 61 + .200\ F$	12.3	.0378	1490	+ 70	+5	.0432	1500	+ 80	+5
	68	1000	915	2460		11.8	.0390	2500	+ 40	+2	.0450	2510	+ 50	+2
	76	1100	1005	2980		11.6	.0405	2910	− 70	−2	.0468	2910	− 70	−2
	96	1310	1190	3930		10.9	.0404	3850	− 80	−2	.0472	3820	−110	−3
	Absolute Saturation..... } $L_\infty = 5000$.					Av. η =	.0396		± 56	±2.6	.0458		± 66	±2.8
	(3) 79 and 91 Complete Periods per Second, $N =$													
79	86	1260	1152	3380	74.5	11.7	.0370	3410	+ 30	+1	.0418	3450	+ 70	+2
91	109	1510	1372	4580	79.3	11.1	.0375	4550	− 30	−1	.0436	4480	−100	−2
	$\rho \sim 56 + .21\ F$.					Av. η =	.0373		± 30	±1	.0427		± 85	±2

F = M. M. F., in ampere-turns per cm.

B = whole magnetic induction, in lines of magnetic force per cm.^{2}

L = metallic magnetic induction, $= B - H$, where $H = \frac{4\pi}{10} F$ is the field-intensity.

H_{obs} = observed value of hysteretic loss, in ergs per cycle and cm.^{3}

ρ = metallic magnetic reluctivity, in thousandths $= \frac{1000\ F}{L}$

μ = magnetic permeability, $= \frac{B}{H} = \frac{10\ B}{4\pi F}$

η_B and η_L respectively = the coefficient of hysteresis, referring to B and L respectively, that is calculated by means of the formulæ:

$$H = \eta_B \left(\frac{B_1 - B_2}{2}\right)^{1.6} \text{ and } H \, \eta_L \left(\frac{L_1 - L_2}{2}\right)^{1.6}$$

H_{calc} = calculated loss by hysteresis, and $\triangle$ = difference between H_{obs} and H_{calc}.

As seen, the magnetic reluctivity varies in the range of tests from $\rho = 72$ to $\rho = 88$.

For M. M. F.'s. of $F \geqq 45$ the observations agree with the law,

$$\rho = \alpha + \sigma F.$$

But the coefficients α and σ are decidedly dependent upon the frequency, increasing with increasing frequency, while the value of absolute magnetic saturation L_∞ decreases with increasing frequency.

The coefficient of hysteresis η is — with the only exception of the one, lowest, reading — constant within the errors of observation, and proves thereby the law of 1.6th power.

But it can not be decided whether H varies with the 1.6th power of B, or of L, since either agrees with the law of 1.6th power, B and L being near enough proportional to bring the differences within the limit of the errors of observation.

Therefore for either value, B and L, the coefficient of hysteresis is calculated and given, η_B and η_L. The coefficient of hysteresis depends decidedly upon the frequency, increasing with increasing frequency. The coefficients of hysteresis are very large, giving hard-steel values.

2. *Magnetometer Tests.*

Table LVII. gives the magnetic characteristic derived from magnetometer tests.

The first two columns give the values found along the stratification, that is in the same condition as the electro-dynamometer tests, with a cross-section of 17.8 $cm.^2$; the first column found by the usual method of reversals, that is by reversing the current repeatedly before each reading; the second column gives the maximum values of magnetization taken from the slow magnetometer cycles in Table LVIII.

TABLE LVII.

MAGNETOMETER TESTS OF IRON FILINGS, MAGNETIC CHARACTERISTICS.

(1) 17.8 cm.^{2} Cross-Section. Along Stratification.						(2) 4 cm.^{2} Cross-Section.			(3) 4 cm.^{2} Cross-Section.		
By Reversals.			By Slow Cycles.			Across Stratification.			Compressed.		
F	L	ρ	F	L	ρ	F	L	ρ	F	L	ρ
12	99	121	32	342	93.7	9	64	140	10	68	147
16	167	96	55	530	103.5	12	91	132	14	112	125
24	262	91.6	90	787	114.5	17	140	114	18	162	111
34	384	$\rho = 77.5 + .375 F$	180	1250	144.0	24	222	$\rho = 106.3 + .384 F$	27	266	$\rho = 97 + .244 F$
44	468					30	260		34	320	
58	574					42	330		45	405	
72	690					54	400		59	512	
95	840					68	532		77	660	
130	1092					105	765		115	932	
145	1150					150	880		160	1165	
170	1220					180	985				
186	1260					210	1150				
200	1310										
225	1370										
Absolute Saturation .... } $L_\infty = 2670$						Absolute Saturation .. } $L_\infty = 2600$			Absolute Saturation .... } $L_\infty = 4100$		

The third column gives the values found across the stratification, with 4 cm.^{2} cross-section. The fourth column gives the tests of iron filings wetted with turpentine and compressed.

Remarkable in all these tests is the considerably higher value of reluctivity, coefficient of hardness and especially coefficient of saturation, and consequently the much lower value of absolute magnetic saturation than that derived from electro-dynamometer tests.

The straight line law,

$$\rho = a + \sigma F.$$

holds for

$$F \geqq 25.$$

The absolute magnetic saturation is very nearly the same across and along the stratification, a little more than half as high as found by the electro-dynamometer method. The magnetic hardness is considerably larger across than along the stratification, 106.3 against 77.5. The compressed iron filings reach a higher value of saturation, but contain more than 30 per cent. of iron.

Table LVIII. gives a number of cycles of these iron filings, and their results, the coefficient of hysteresis η being given for B as well as for L.

TABLE LVIII.

MAGNETOMETER TESTS OF IRON FILINGS, HYSTERETIC CYCLES.

	(I.) 17.8 cm.² Cross-Section.								(II.) 4 cm.² Cross-Section.				(III.) Compressed. 4 cm² Cross-Section.	
	(1)		(2)		(3)		(4)		(1)		(2)		(1)	
F	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r	L_d	L_r
180	±1250													
170	1220	1210												
160	1190	1168												
150	1162	1123												
140	1127	1073												
130	1090	1020												
120	1050	960												
110	1010	900												
100	964	837												
90	917	770	±787						±660				[±75]	
80	868	700	750	716					620	600			±650	
70	812	620	704	640					580	530			628	607
60	757	540	650	554	[±55]				540	460			590	538
50	695	460	596	472	±530				490	392	±390		548	460
40	630	370	540	388	462	400	[±32]		450	320	360	330	493	375
30	560	280	480	296	408	305	±342		400	230	320	260	427	280
20	476	145	406	170	340	180	280	190	340	130	280	160	350	165
10	370	—55	310	—[illegible]0	260	10	200	30	250	0	210	40	255	0
0	±240		±1[illegible]		±150		±100		±140		±100		±150	
$H =$ $L =$	6034 1250		2820 787		1480 530		738 324		2300 660		960 390		2.022 650	
$\eta_L =$	.0669		.0656		.0648		.0651		.0709		.0686		.0639	
	Av. η_L = .0656								.0698				.0639	
$B =$	1475		900		600		382		770		450		744	
$\eta_B =$	.0514		.0541		.0531		.0545		.0554		.0546		.0514	
	Av. η_B = .0533								.0550				.0514	

These coefficients η are larger than the values found by electro-dynamometer tests.

Table LIX. gives a collection of the different values of the magnetic constants of these iron filings, a, σ, L_∞, η_L, and η_B, as found for the material proper (Chapter I.), for the filings by electro-dynamometer tests along stratification, for the frequencies of 180, 114, and about 85 complete periods per second; by magnetometer tests along and across stratification, and compressed.

Fig. 23 gives the different magnetic characteristics, with the air line as dotted line. Fig. 24 gives the different curves of hysteresis, the observed values being marked by crosses, and Fig. 25 gives the four magnetometer cycles of hysteresis, from Table LVIII., 1.

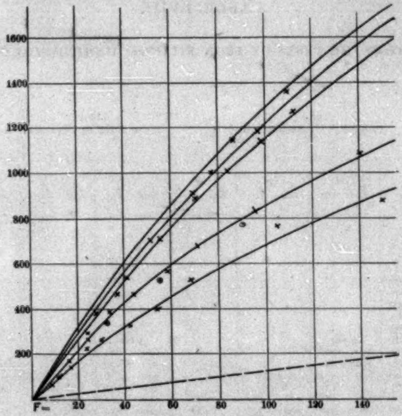


Fig. 23.—Iron Filings. Magnetic Characteristics.

TABLE LIX.

MAGNETIC CONSTANTS OF IRON FILINGS, 30 VOLUME PER CENT.

	Number of Complete Cycles per Second	Coefficient of Magnetic		Absolute Saturation	For	Coefficient of Magnetic Hysteresis	
		Hardness	Saturation				
	N	α	σ	L_∞	$F \gtrsim$	η_L	η_B
The Sheet-Iron proper ..	67~170	.275	.058	17.24	~ 8	.0035	.0035
Filings, 13.7 cm.^{2} cross-sec.	180	64	.218	4.59	~45	.0533	.0457
" " "	114	61	.200	5.00	~45	.0458	.0396
" " "	~85	56	.21	4.76		.0427	.0373
" 17.8cm.^{2} Magnetom'r.	Very Slow.	77.5	.375	2.67	~30	.0656	.0533
" 4cm.^{2}, across stratifi'n	"	106.3	.384	2.60	~20	.0698	.0550
" 4 cm.^{2}, compressed..	"	97	.244	4.10	~25	.0639	.0514

Herefrom it seems, that α, σ and η are largest for very slow magnetic cycles, as in the magnetometer tests, *decrease* for *increasing* frequency, reach a *minimum* for a moderate frequency,

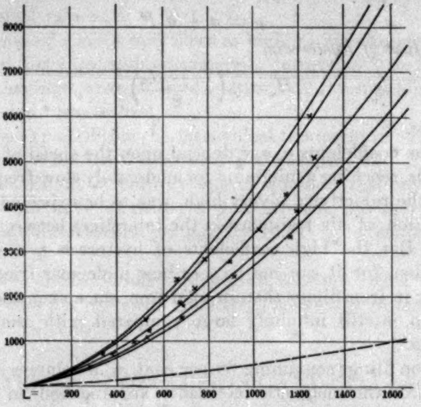


FIG. 24.—Iron Filings. Curves of Hysteresis.

and *increase* again for *increasing* frequency, though being at the frequency 180, still far lower than for slow cycles.

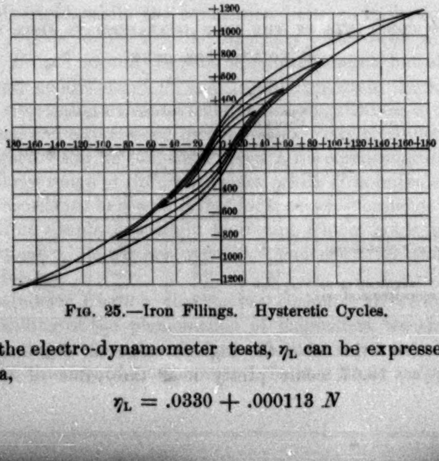


FIG. 25.—Iron Filings. Hysteretic Cycles.

For the electro-dynamometer tests, η_L can be expressed by the formula,

$$\eta_L = .0330 + .000113\ N$$

The conclusions derived herefrom are,

"*Even for such heterogeneous materials as iron filing the linear law of reluctivity,*

$$\rho = a + \sigma F$$

and the law of hysteresis,

$$H = \eta \left(\frac{L_1 - L_2}{2} \right)^{1.6}$$

hold true,

But the coefficients a, σ, η depend upon the speed of magnetic variations, reaching a minimum for moderately slow frequencies."

That the reluctivity is very high was to be expected from the introduction of air resistance in the interstices between the iron filings. But the high coefficients of hysteresis η need still an explanation, for it can not be seen how molecular friction could be larger in iron filings than in solid iron, since even the smallest iron chip is still infinitely large compared with the sizes of molecules.

The iron filings containing 30 per cent. = .3 volumes of iron, in Table LX. the magnetic constants are reduced to the iron proper by multiplying a and σ, and dividing L, L_∞ and H by .3, consequently multiplying $\eta = \frac{H}{L^{1.6}}$ by $.3^{6} = .486$.

TABLE LX.

MAGNETIC CONSTANTS OF THE IRON CONTAINED IN IRON FILINGS, 30 VOLUME PER CENT.

	Number of Complete Cycles per Second	Coefficient of Magnetic			Absolute Saturation
		Hardness	Saturation	Hysteresis	
	N	a	σ	η_L	L_∞
The Sheet-Iron proper	67~170	.275	.0558	.0035	17.24
Filings, 13.7 cm.^{2} cross-section.	180	19.2	.0654	.0259	15.30
" " "	114	18.3	.0600	.0222	16.67
" " "	~85	16.8	.0630	.0207	15.87
" 17.8 cm.^{2}, magnetom'r tests	Very Slow.	23.2	.1125	.0318	8.90
" 4 cm.^{2}, across stratification.	"	31.9	.1152	.0339	8.67

As seen from this table, the highest values of absolute saturation $L_\infty = 16.67$, come pretty near the value of the iron proper, 17.24; but the values derived from magnetometer tests remain far below that.

But even the lowest values of the coefficient of hysteresis η are still hard-steel values.

The values of η are 4 to 6 times as high as the highest values ever found for sheet-iron (commercial ferrotype) 7 to 11 times as high as average wrought-iron, and 10 to 17 times as high as the lowest wrought-iron values.

It is to be expected that the mechanical treatment in cutting the iron filings has increased their magnetic hardness and hysteresis somewhat. But it is entirely out of question that mechanical treatment can have increased η 7 to 11-fold, the more as the value of absolute saturation $L_\infty = 16.67$ is in contradiction thereto.

The only conclusion left is, therefore, *that the looped curve of hysteresis does not represent the energy consumed in the iron by molecular friction.*

CHAPTER. V.—CONCLUSIONS AND FALLACIES.

The tests communicated in the former chapters seem to prove that molecular friction in magnetizable materials under variations of magnetization is much more constant a phenomenon than has been usually supposed. The connection between loss by molecular friction H and amplitude of induction L seems to be absolutely rigid, while the connection between induction L and M. M. F. F is decidedly flexible, especially with lower M. M. F.'s, because L does not only depend upon the present, but also upon the former conditions of F and L and even upon the time by a kind of viscous hysteresis or better called sluggishness as observed by Ewing, and also noticed by me under certain circumstances on the magnetometer, so that for a given L the corresponding F can have a large range of different values, while H is univalent.

In concordance herewith is that for the correspondence between L and F no simple law could be found which holds over the whole range, while the law of interdependence of L and H evidently does so.

Consequently I believe that the best chance to arrive at a fuller understanding of the phenomenon of magnetism we shall have when starting in the research from the correspondence H—L. However, this law of 1.6th power I believe is not a *differential* law, like for instance the quadratic law of gravitation, but in an *intregal* law like the law of *probability* with which it seems to be connected in some way.

In the former chapters we have for the determination of the *molecular friction* made use largely of the cyclic curve of hysteresis, that is the correspondence between the magnetic induction and the M. M. F. when the latters performs a complete cycle.

If the magnetization is given as magnetic intensity or moment,

$$\mathrm{I} = \frac{L}{4\,\pi} = \varkappa\, H,$$

and the M. M. F. as field intensity H, the area of this loop directly represents the energy expended by the variation of the M. M. F., in ergs per cm.^{3} and cycle.

If the magnetization is given as magnetic induction, L or B, the M. M. F. as field intensity, H, the area has to be divided by $4\,\pi$, to give the energy. But if the M. M. F. is given in current-turns per cm., the area is equal again to the consumption of energy, in ergs, or, if the M. M. F. is given in ampere turns per cm., F, since 1 ampere $= 10^{-1}$ absolute units; the area is 10 times the energy in ergs. This is another reason why I preferred the use of ampere turns per cm., F, as M. M. F., to make this area directly equal to the hysteretic energy, with a power of ten as factor, as usual in our system of practical units.

Giving L in volt lines, F in ampere turns, the area is directly equal to H in volt seconds or joules.

As said before, this looped curve of hysteresis measures the energy expended by the M. M. F. during a complete cycle.

It has been assumed then, that the area of this loop represents the energy consumed by molecular friction in the iron. This is a *fallacy*. The area of this looped curve is not the energy dissipated by molecular friction in the iron. Warburg and Ewing have shown—the former by supposing the cycle of M. M. F. performed by changes in the position of steel magnets, and determining the energy expended in performing these changes in position; Ewing by supposing the magnetic cycle produced by a cyclic variation of the exciting current in a magnetizing helix and calculating the energy consumed by the E. M. F.'s induced in the magnetizing helix by the cyclic variation of magnetic induction, that the energy expended by the M. M. F. during a complete cycle is equal to the area of this looped curve.

Hence, it has been concluded that the area of this loop represents the energy expended by molecular friction in the iron.

Here is the mistake in the conclusion. For

"*The area of the looped curve of hysteresis represents the energy dissipated by molecular magnetic friction then, and only then, when during the magnetic cycle neither energy is exerted upon the magnetic circuit by another source of energy, nor work done by or in the magnetic circuit.*"

Instances of the first case have been observed—and misinterpreted—numerously.

For instance, on pages 114–115, and on pages 319–320 in Ewing's book is shown, that under the influence of vigorous vibration, or of an alternating current passing lengthwise, that is in the direction of the magnetic flux, through the magnetized wire, the looped curve of hysteresis more or less collapses, *hysteresis disappears.* But *not so molecular friction.* The energy dissipated by molecular friction is simply derived not from the cyclic varying M. M. F., but from the *force vibrating the wire*, viz: from the *alternating current.* For when violently vibrating a magnetized body molecular motions are produced by the mechanical force, which consume a part of its mechanical energy. But the best proof is, that under circumstances, by the action of such a mechanical force, the magnetic loop made by the correspondence of *L* to *F* can be overturned, so that the rising curve of magnetization is higher than the decreasing, that is the cycle represents, not expenditure, but production of energy. Since obviously molecular friction can not produce energy, here the action of mechanical force is plain.

In rotating the keeper before the poles of an electromagnet, magnetism and magnetizing current are made fluctuating, and in plotting the magnetism as a function of the M. M. F., we derive such an overturned loop.

To such overturned loops, based on actual tests made on an alternating dynamo of the "humming bird" type, are shown in Fig. 26.

Here simple mechanical energy has delivered not only the energy dissipated by molecular friction in the iron, but also the energy exerted by the varying magnetion upon the M. M. F., and while the M. M. F. does not expend, but receives energy, the mechanical force of rotation expends energy. Consequently, if the magnet is not an electro-magnet, but a steel-magnet, it will be strengthened, as is well known. Another instance is, if we alternately tear the keeper off a permanent magnet and put it on again. After a number of cycles the permanent magnet will come into a stationary condition, neither lose nor gain in magnetic potential. Nevertheless, by molecular friction in these parts of the steel magnet, and in the keeper, where the magnetism varies in strength and direction, energy is dissipated. This is derived consequently, from the source of mechanical energy. The case may be similar in dynamo-armatures. The opposite phenomenon, that the hysteretic loop represents more energy than expended by molecular friction, is still more frequent.

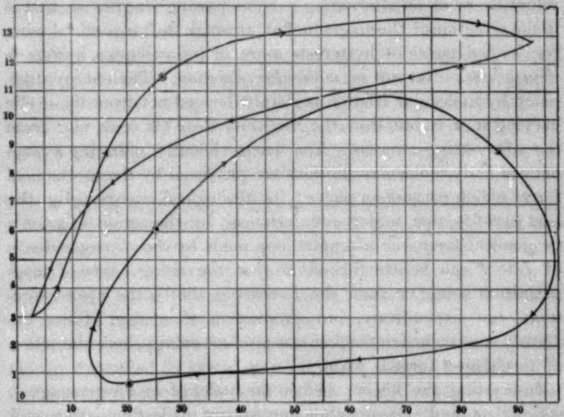


FIG. 26.—Overturned Hysteresis Loops of "Humming Bird."

For instance, if eddy, or Foucault-currents are induced in the iron, the hysteretic loop is considerably widened, and represents now not only the energy expended by molecular friction, but also the energy spent by the eddies.

But since the eddy currents are electric currents also, and represent a certain M. M. F., in this case the difficulty is overcome by stating that not the impressed M. M. F., but the M. M. F. resulting from the impressed M. M. F. and the M. M. F. of eddy currents has to be considered in determining the energy spent by molecular friction. This is still more plain in the case of the transformer, where it evidently would be incorrect to represent the induction L only as a function of the impressed M. M. F. of primary current, instead of the resultant M. M. F. of primary and secondary current.

But in the magnetic circuit built up of iron filings, as treated in Chapter IV., III., we have a case where without the existence of secondary currents the hysteresis loop represents more energy than spent by molecular friction.

In this case evidently mechanical motions take place in the iron filings, which consume energy, derived from the M. M. F.

The mechanism of action may be about the following :—When the M. M. F. increases, more and more iron filings fall in alignment, by setting up chains of filings as soon as the M. M. F. is large enough to cause the motions required hereto. When the M. M. F. decreases, these chains of filings will be maintained down to a much lower M. M. F. than was required to produce them. The consequence hereof is that—independent of molecular hysteresis —for the M. M. F. on the decreasing branch the apparent magnetic reluctivity will be considerably smaller, hence the induction larger, than for the same M. M. F. on the increasing branch—that means, the hysteric loop will be widened, and widened by that amount of energy expended by the mechanical motions of the iron filings.

The same is seen in the case of the loose wire spirals, in Chapter IV., I., where the increasing M. M. F. brings the spirals in closer contact, while in the case of the crosswise laminated iron, Chapter IV., II, no such expenditure of energy is possible, and, indeed, experiment gives a much closer agreement between the hysteretic loss of the cross laminated iron and that of the material proper, the difference being small enough to be explained by the unequalities of magnetic distribution.

To test the correctness of this reasoning, I dipped the tube containing the iron filings in melted paraffin. After having cooled down, I made another set of tests, of the hysteretic cycles of these iron filings (magnetometer tests, along stratification) and got the values:

$\pm F$:	$\pm L$	H:	η:
87	892	2606	.04959
50	616	1122	.03860
31	424	520	.03252

while the tests of these iron filings without paraffin had given: $\eta = .0656$.

The tests show a considerable decrease of the value of η, when the filings were hindered in their motion by filling the interstices with paraffin, especially for lower M. M. F's, and thereby prove the assumption.

These tests were made on a hot summer day, and the still comparatively large values of η seem to indicate, that motions of the filings still took place, especially under larger magnetic strains, that is, that with a M. M. F. $F = 87$ the paraffin partly gave way before the push of the iron filings, at the same time these tests prove conclusively, that the value η decreases, if motions of the iron filings are impeded, as was to be expected.

The simplest case of this phenomenon is that of an electromagnet with keeper excited by a slowly alternating current, at a certain M. M. F. the keeper will be attached, and then held down to a far lower M. M. F. since a much larger M. M. F. is required to attract the keeper over a distance, than is required to keep it in contact. Consequently the loop performed by such an electromagnet will not represent the molecular friction only, but this molecular friction plus the mechanical work done by the magnet.

In the alternating current synchronous motor with wireless shuttle armature the whole mechanical energy is derived by an enlargement of the cyclic curve of magnetization of the field magnet.

Very likely in the amalgam of iron we have such a case also.

An interesting fact is then, that the law of the 1.6th power holds for iron filings also, and consequently the expenditure of mechanical energy in the motions of the iron filings must follow the same law, and nevertheless these iron filings do not resemble at all the conditions claimed for the molecules of paramagnetic substances. For these iron filings are neither permanent magnets, nor are their distances infinitely large compared with their dimensions, as must be assumed for molecules.

This explains also, why the coefficient η is largest for very slow cycles, decreases, and after reaching a minimum for a moderate frequency, increases again. This explains also the corresponding variation of absolute saturation.

That, nevertheless, the law of the 1.6th power holds, proves, that this law does not depend upon a particular constitution of the material, but is of more general meaning.

Another consequence is. if, as we have seen, by mechanical vibrations the hysteretic loop is made to collapse, this does not mean, that by shaping the magnetic circuit so that the alternating magnetism produces vibration, the loss of energy by molecular friction would be avoided or overcome, as has been thought by misinterpretation of the tests referred to above, but in the contrary such an arrangement would have just the opposite effect, to add to the unavoidable loss by molecular friction the loss by mechanical vibration. It is not yet proved, indeed, that under the influence of mechanical vibration, or of an alternating longitudinal current the molecular friction is still the same, although this is made very likely by all that we know about the constancy of this molecular friction. Further tests will give more light upon this matter.

It is highly probable, that the initial inward bend of the magnetic characteristic, and the deviation of the metallic reluctivity from the linear law, caused thereby, is merely due to the expenditure of energy by the M. M. F. for molecular friction, and that consequently, if the energy of molecular friction is derived from another source, for instance mechanical vibration, the magnetic reluctivity follows the linear law from the beginning, as observed by Ewing, and the inward bend of the magnetic characteristic dissapears.

This explains the enormous increase of permeability for low M. M. F.'s., caused by vibration. In the absence of an external source of energy the rise of magnetic induction following the linear law of reluctivity is for low M. M. F.'s. made impossible by the fact, that in this case more energy must be expended by molecular friction, than would be derived from the M. M. F. by the E. M. F. induced in the exciting circuit.

THEORY OF MOLECULAR MAGNETS.

Relatively the best explanation of the phenomena of magnettic induction and of magnetic hysteresis is afforded by the assumption, that the molecules of the paramagnetic materials are permanent magnets, which, as long as no outside directing force H acts, have no definite direction and consequently no resulting magnetic moment, but, following their mutual attraction, are grouped in pairs and chains.

By the application of an outside force H, the molecules are turned into alignment with *H*, against the opposing forces of mutual attraction, and hereby deflected by a certain angle. Now for certain positions of molecules exists an angle of deflection, which makes *H* a maximum, so that for a further increase of the angle of deflection a smaller value of *H* is required, and consequently the *H* necessary to reach this critical angle of deflection overthrows the molecule—an irreversible process, which represents the loss of energy by what is called molecular friction.

This theory can not, indeed, be considered an explanation of the phenomenon of *magnetism*, since it refers it back to permanent molecular *magnetism* again; it is merely an explanation of the particular shape of the magnetic characteristic and of the loss by molecular friction.

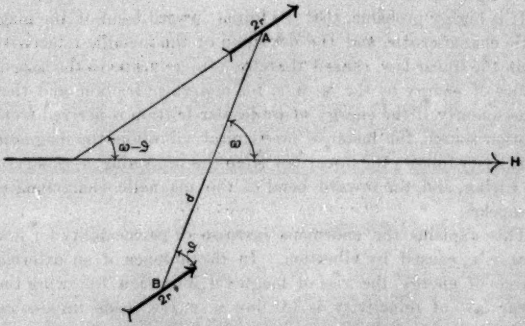


Fig. 27.—Theory of Molecular Magnets.

This theory of molecular magnets, and the unstable equilibrium reached by them for a certain *H*, has been worked out especially by Ewing. But in determining the fundamental equation of this theory, the equation of equilibrium of a pair of molecules acted upon by an outside force *H*, Ewing makes an assumption which is in contradiction to all our present knowledge of molecular physics.

All the facts of the kinetic theory of gases, of thermodynamics, etc., carry to the conclusion, that the *dimensions* of molecules are *infinitely* small compared with their *distances*.

But Ewing supposes the distance of the centres of molecular magnets is not much greater than their length, to be able to make the assumption, that the attracting force between the magnet poles pointing away from each other is negligibly small compared with the attraction of the poles pointing towards each other- while both forces become nearly *equal* by assuming the distance of the molecules very large compared with their dimensions. Ewing's assumption introduces a quadratic term into the equa, tions, where we must get a cubic term, and essentially changes the conditions of unstable equilibrium.

It would carry me too far for the scope of this paper, to give a complete essay on the theory of molecular magnets, and so I must leave this for a future paper and give only the general way of conclusions.

Let, in Fig. 27, represent

H = the direction and intensity of the M. M. F. (field intensity);
A and B, two molecules, being permanent magnets;
d = the distance of the centres of the two molecular magnets;
$2r$ = the distance of the poles of each of the two molecular magnets; (1)
m = the pole-strength of the molecular magnets;
ω = the angle between the distance d of the centres of molecules and the M. M. F. H;
ϑ = the angle of deflection of the molecular magnets.

We have, then—

Deflecting couple,

$$M = -2\,r\,m\,H \sin(\omega - \vartheta).$$

Restoring couple,

$$N = \frac{8\,r^2\,m^2 \sin\vartheta\cos\vartheta}{d^3}.$$

Consequently,

Conditions of equilibrium,

$$M + N = 0, \qquad \text{or}$$

$$\frac{4\,r\,m}{d^3}\sin\vartheta\cos\vartheta = H\sin(\omega - \vartheta). \tag{2}$$

The fundamental equation of a pair of molecular magnets. Denoting

$$\frac{4\,r\,m}{d^3} = \lambda, \tag{3}$$

we derive

$$\frac{\sin \omega}{\sin \vartheta} - \frac{\cos \omega}{\cos \vartheta} = \frac{\lambda}{H} \,. \tag{4}$$

As condition of *unstable equilibrium*, we get from

$$\frac{d\,H}{d\,\vartheta} = 0 \qquad \text{the equation}$$

$$\left.\begin{aligned} &\tan \omega + \tan^3 \vartheta_0 = 0, \qquad \text{or} \\ &\tan \vartheta_0 = - \sqrt[3]{\tan \omega}, \end{aligned}\right\} \tag{5}$$

and, herefrom

$$H_0 = \frac{\vartheta}{\left\{ \sqrt[3]{\sin^2 \omega} + \sqrt[3]{\cos^2 \omega} \right\}^{\frac{3}{2}}_{\mathrm{si}}} \,.$$

As *minimum value of* H_0, which causes unstable equilibrium, we get

$$H_0 = \frac{\lambda}{2} \qquad \text{for} \qquad \omega = 135^\circ$$

As *maximum value of* H_0, we get

$$H_0 = \lambda \qquad \text{for} \qquad \omega = 90^\circ, 180. \tag{7}$$

Equation (5) gives the condition of unstable equilibrium,

$$\omega > 90^\circ. \tag{8}$$

From these equations, we see now

"These pairs of molecules, which in their initial position make a sharp angle, $\omega < 90^\circ$, with the M. M. F. H, never reach unstable equilibrium; these pairs of molecules, which in their initial position make an obtuse angle, $\omega > 90^\circ$, with the M. M. F. H, reach unstable equilibrium between the values of M. M. F. $H_0 = \frac{\lambda}{2}$ and $H_0 = \lambda$, and the instability is reached first for the angle $\omega = 135^\circ$, last for $\omega = 90^\circ$ and $\omega = 180^\circ$. At this point of instability the molecules are overturned and pass by an irreversible motion—which causes the dissipation of energy into heat—in the position corresponding to the angle $\omega' = 180 - \omega$."

A complete view of these phenomena can be had best geometrically.

Considering, in a system of polar co-ordinates,

$$\left.\begin{aligned} &H \text{ as radius vector,} \\ &\omega \text{ as amplitude,} \end{aligned}\right\} \tag{9}$$

the equation (6) represents the *sextic hypocycloide*, with λ as half-axis, a curve enveloped by a straight line of constant length λ sliding within a right angle.

This curve, in rectangular co-ordinatesof the equation.

$$\sqrt[3]{x^2} + \sqrt[3]{y^2} = \sqrt[3]{\lambda^2} \qquad (10)$$

is given in Fig. 28.

It is at the same time the "Evolute of the Circle."

Only the arc in the second quadrant is of interest to us, and drawn in Fig. 29, while the arc in the first quadrant is dotted.

Set, in Fig. 29, $O\,A$ = the direction of the two molecules A and B in their initial position.

Draw the M. M. F. $\overline{O\,H} = H$ under its angle $\omega = H\;O\;A$ and lay from H the tangent $\overline{H\,T}$ on to the hypocycloide, than $\overline{T\,H}$ is the direction of the molecules when deflected by M. M. F. H., and angle $H\;C\;O = \vartheta$.

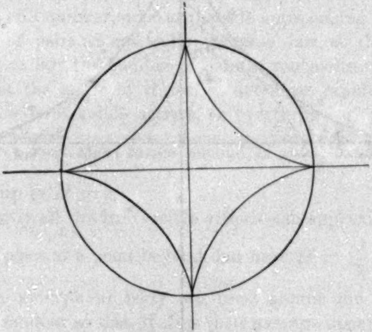


FIG. 28.—Sextic Hypocycloide.

In the first quadrant, $\omega < 90°$, we see that when H increases from zero to infinite, angle ϑ steadily increases from 0 to ω, also the direction of the molecules varies steadily from $\overline{O\,A}$ to $\overline{O\,H}$.

In the second quadrant, $\omega > 90°$, if H increases from zero to infinite the angle ϑ increases from 0 to a maximum value ϑ_0, which is reached at the point of intersection H_0 of the M. M. F. H with the hypocycloide.

In this point H_0 the tangent $\overline{H\,T}$ ceases to exist, instability is reached, and the angle ϑ abruptly varies from the value ϑ_0 to the value ϑ_0^1, the molecules are overthrown from the direction $\overline{C_0\,H_0}$ to the direction $\overline{C_0^1\,H_0}$, by the angle of hysteresis,

$$C_0\,H_0\,C_0^1 = \varphi_0.$$

For a farther increase of H the angle ϑ increases again by the variation of the tangent laid on to the dotted curve.

Withdrawing the M. M. F. H again, it no longer intersects the hypocycloide, since now, after the overthrow, the dotted curve is in use, and consequently no instability is reached.

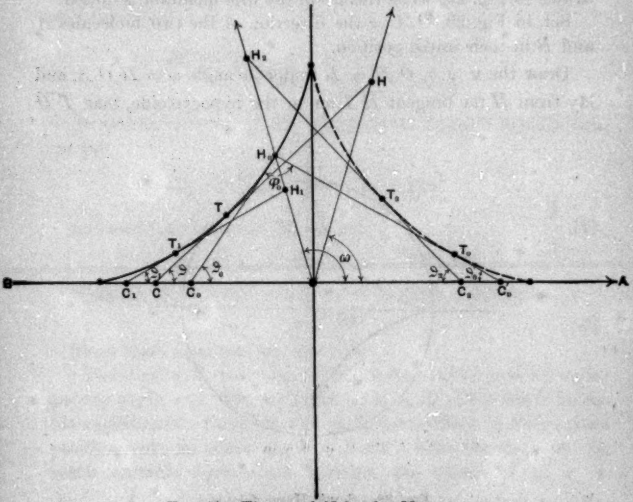


FIG. 29.—Theory of Molecular Magnets.

Considering now the phenomena taking place by the action of a M. M. F. H of given direction, we see first, that the number of pairs of molecules with a given ω is proportional to $\sin\omega$, so that very few pairs of molecules exist with angles of nearly $= 0$ or 180°, the number of pair increases first rapidly, last very slowly for increasing ω, and reaches a maximum for $\omega = 90^\circ$.

For increasing H we see that first no irreversible motions take place, and the magnetic moment—the projection of the molecular moments upon the direction of M. M. F.—increases slowly, until $H = \frac{\lambda}{2}$ is reached. Between $H = \frac{\lambda}{2}$ and $H = \lambda$ all the irreversible action take place, and the magnetism increases rapidly, and very slowly again, after $H = \lambda$ is passed.

But since at $H = \frac{\lambda}{2}$ the circle drawn with H as radius, touches the hypocycloide, at $\omega = 135°$, a small increase of H causes a great number of pairs of molecules to be overthrown, and the magnetic moment and the hysteretic loss to increase very rapidly. But very soon, for increasing H, the circle drawn with H as radius intersects the hypocycloide under a steeper and steeper angle, the increase of the range of overthrow of ω decreases, and the rapidity of increase of magnetization and hysteresis decreases still quicker, since at the side approaching $\omega = 180°$ the number of pairs of molecules decrease fast, at the side approaching $\omega = 90°$ the number of pairs of molecules still slowly increases, but the angle of throw φ_0 decreases rapidly, so that almost all the irreversible actions, or overthrows of molecules, take place in a very short range beyond $H = \frac{\lambda}{2}$, and very few afterwards, up to $H = \lambda$.

Consequently all the irreversible actions can approximately be said to take place at a point beyond, but near $H = \frac{\lambda}{2}$.

Now is an amorphous body we must assume the molecules scattered at random so that, if D is their average distance, this is the distance of molecules existing most frequently, but all the other distances between pairs of molecules exist, in a frequency determined by a *law of probability*, and consequently, if

$$\Lambda = \frac{4\,rm}{D^3} \text{ is the value of } \lambda = \frac{4\,rm}{d^3}$$

corresponding to the average distance D of molecules, this $\lambda = \Lambda$ is the most frequent value, but all the values of λ exist, though rapidly becoming less frequent, the more they differ from $\lambda = \Lambda$.

For a given λ, the greatest part of the magnetic induction and the hysteretic loss takes place at or near a point $H = \frac{\lambda}{2}$.

Consequently, in a body as a whole the greatest part of the magnetic induction and the loss by molecular friction simply depend upon that *law of probability* which determines the distances of molecules.

This conclusion is, indeed, derived under the assumption that the molecules act upon each other only in pairs. To consider the more general case of mutual action of more than two molecules, would carry me too far here, the more as the assumption of an arrangement in pairs can not be so far from giving an approximately true picture of the phenomenon, since the mutual action depends upon the third power of distance, and consequently only the next molecule will have a greater influence.

As conclusion, we derive, then,

"*In first approximation, the magnetic induction and the molecular friction depend upon the* M. M. F. *by the law of probability of molecular distances.*"

The point of maximum increase of induction is not the same as the point of maximum increase of molecular friction, since different factors enter into the function of probability.

The *law of hysteresis*, of the 1.6th power, is the interdependence of two functions depending upon the same law of probability, hence can be of simpler form than either function.

A more complete research on these theoretical questions, I must postpone for a later occasion.

Eickemeyer Laboratory, Yonkers, N. Y., July, 1892.

APPENDIX.

1. As *Methods of Determination*, I have generally used the Electro-dynamometer method and the Eickemeyer Differential Magnetometer; using the ballistic method but a few times for controlling observations, generally employing in this case an electro-dynamometer with separately excited fixed coil as ballistic galvanometer.

The electro-dynamometer-method has the advantage of great sensitiveness and large range of readings (by varying the additional resistance), and is the only method which determines the Foucault —or eddy—currents also, in using alternating currents, its results being specially applicable for alternate-current practice. But it is limited in so far as it can be used with laminated materials only, and has the serious disadvantage of giving too small values of M. M. F. for higher saturations, so that for the determination of the magnetic characteristic it can be used only for low and medium magnetizations, while at higher saturations the wave of the current is more and more changed in shape, and becomes pointed, so that the maximum value of current is occasionally very many times higher than $\sqrt{2} \times$ the effective value. and consequently can not be calculated therefrom. Because of this feature of the method, in Fig. 2 and 3 of my former paper on hysteresis, the values of M. M. F. beyond $B = 17{,}000$ are given to small, being calculated from the "effective" electro-dynamometer readings as explained there. Therefore for reluctance determinations at higher saturations I abandoned the electro-dynamometer-method altogether, and used the magnetometer or ballistic method.

The ballistic method has the largest range of readings, and is applicable for any shape of test-pieces. But it has the disagreeable feature of instantaneous readings, and, in cyclic tests, the readings depend upon the exactness of former readings, so that the errors of observation are summed up. But the greatest objection to the ballistic method is, that it records only the instantaneous changes of magnetization, but fails to take account of the so-called "magnetic creeping," wrongfully called "time hysteresis," the phenomenon that on the unstable branch of the magnetic characteristic the magnetism does not increase suddenly with the increase of M. M. F., but after a smaller increase simultaneously with the increase of M. M. F., the magnetism continues to rise still for seconds and minutes. This slow rise is not recorded, and in consequence thereof the ballistic tests by the step-by-step method usually yield too low values of induction, while the method of reversals gives correct results at least for the higher values of induction.

In consequence of this creeping, with cast iron under certain conditions, the ballistic galvanometer may give a larger throw than with soft wrought iron.

The usual magnetometer method takes account of this magnetic creeping—in fact, this phenomenon has been observed by the magnetometer method (Ewing, p. 121 *et seq*); but the magnetometer is not applicable to closed magnetic circuits.

The Differential Magnetometer has the great advantage of being a zero method, yet I could not make it applicable for very low magnetizations.

2. *Demagnetizing by alternating currents, and screening effect of eddies:* It has been asserted repeatedly, that the best means of destroying remanent and permanent magnetism is the application of a rapidly alternating M. M. F., while again other experimenters failed to succeed in demagnetizing by alternating currents.

It is beyond doubt that a rapidly alternating magnetic induction leaves under normal circumstances not only no trace of remanent magnetism, but after being exposed to such an alternating magnetic induction, remanent or permanent magnetism, which before existed in the iron, are found destroyed.

But to do this, the alternating magnetic field must be powerful enough to magnetize the iron through. For the eddy-currents induced in the iron by the alternating magnetism represent a true M. M. F. also, which combines with the impressed M. M. F. so that the resulting M. M. F. in the interior of the sample may be very small or almost nil, in spite of a large impressed M. M. F. To calculate this "screening effect" of eddy-currents, we cannot assume the permeability of the iron as constant, as usually done in this case. But in another way we can determine the maximum possible M. M. F. of eddy-currents, and therefrom derive the minimum impressed M. M. F., which is sure to demagnetize the sample.

By assuming the sample magnetized by the impressed M. M. F. up to absolute saturation L_∞, we can calculate therefrom the E. M. F. and thence the eddies set up thereby, and their M. M. F.

Let us suppose the sample to be a rod or ring of circular cross-section, with radius R.

Let L_∞ be the absolute saturation attainable by the material of the sample, κ its specific electric conductivity, N the frequency of the alternating impressed M. M. F.

A cylindrical zone of thickness dr and radius r (and unit width) then incloses the magnetic flux,

$$m = r^2 \pi L_\infty ,$$

as maximum and, consequently, in this zone is induced an E. M. F.,

$$\begin{aligned} e &= 2 \pi M N 10^{-8}, \\ &= 2 \pi^2 r^2 L_\infty N 10^{-8}, \end{aligned}$$

and, since the electric conductivity of this cylindrical zone is

$$k = \frac{\kappa\, d r}{2 \pi r},$$

the electric current induced in this zone is

$$dc = e\,k$$
$$= \varkappa \pi r L_\infty N 10^{-8}\, dr,$$

and consequently the current induced in the rod per unit length, that is, the induced M. M. F., or M. M. F. of eddy-currents, is, in maximo,

$$f = \int_0^R dc,$$

$$= \varkappa \pi L_\infty N 10^{-8} \int_0^R r\, dr,$$

$$f = \frac{\varkappa \pi R^2 L_\infty N 10^{-8}}{2} \text{ ampere-turns per cm.}$$

Now, suppose the sample to have 1 cm.^{2} cross-section, that is,

$$R = .57, \quad \text{and let}$$
$$\varkappa = 30{,}000,$$
$$L_\infty = 16{,}000,$$
$$N = 100,$$

then we derive

$$f = 240 \text{ ampere-turns per cm.,}$$

as maximum value of induced M. M. F., which it would reach if the whole sample were magnetized up to absolute saturation, and no difference of phase exists between the different zones.

Hence, if the maximum impressed M. M. F. is

$$F = 250,$$

which combines with the induced M. M. F. f 240 to the resulting M. M. F. F_0; this resulting M. M. F. is

$$F_0 = \sqrt{F^2 - f^2} = 70 \text{ ampere-turns per cm.,}$$

since f lags behind F_0 by one-quarter period. Consequently, $F = 250$ may not be sufficient to quickly destroy the permanent magnetism. It will destroy it, however, after a short time, since the sample gets heated by the eddy-current and its electric conductivity $\varkappa$ thereby decreases. So an increase of temperature up to 200° C. will decrease the conductivity by about 33 per cent., to $\varkappa = 20{,}000$, and then we get

$$f = 160\,;$$

$$\text{consequently,} \quad F_0 = 190 \text{ ampere-turns per cm.,}$$

sufficient to destroy any permanent magnetism, except, perhaps, in glass-hard materials.

With regard to these induced or eddy-currents, which circulate in laminated materials also, though in a lesser degree, upon the magnetic characteristic, when determined by the electro-dynamometer method, they will generally have no perceptible influence, since they lag one-quarter of a period behind and consequently change the resulting M. M. F. very little. Only at very high frequencies with thicker sheet-iron, the magnetic reluctance becomes apparently increased somewhat for lower and medium magnetizations, by the demagnetizing effect of the eddies, while at higher saturations the influence of eddies upon the characteristic entirely disappears even in thick sheets and for high frequencies.

3. *Denotations*:—In the foregoing, I have used the terms "cast-iron," "steel," "wrought-iron," though these terms have, nowadays, scarcely any individual meaning, either mechanically or magnetically. Mechanically, since large varieties of cast-steel are rolled, drawn into wire, etc., and thereby have assumed fibrous textures, while wrought-iron is cast in the mitis-iron, and thereby formed into an homeogeneous material, and the different kinds of cast-steel completely overbridge the gap between cast-iron and soft, tough material.

Magnetically, some kinds of cast-steel are identical with soft wrought-iron; others approach cast-iron, so that the difference between wrought-iron, steel and cast-iron does not exist, and, if we intended to classify the materials—so far as they are contained in the tests given in the paper, we would distinguish about four classes.

1. Soft material, a low, below 1, σ low, below .06.
2. Medium hard material, a medium, from 1 to 3, σ low, below .07.
3. Low permeability, a medium, from 1 to 3, σ high, beyond .09.
4. Hard material, a high, beyond 3.

In the first class range Norway-iron, sheet-iron, soft wire, annealed cast-steel, mitis metal.

In the second class cast-steel, welded-steel, etc.

In the third class some cast-steel and cast-iron.

In the fourth class glass-hard steel, magnet-steel.

4. *Chemical Analysis*:—It may be considered as of some interest to give the chemical analysis of the tested samples, and I originally intended to analyze them, but had to give up this idea because of the enormous time necessary for an exact analysis and more especially as I came to the conclusion that a chemical analysis would be of a very doubtful, if of any value For, as I have shown in the fore going, the magnetic constants of materials depend much more upon their physical than upon their chemical constitution, so that a chemical analysis can be of value only, if the *physical* and *mechanical* properties of the material, and its *history* is given also, the latter in so far, as chemical constituents influence the material considerably even if they do not exist any more in the finished material, by the changes brought about by their entering and leaving the material, as it seems to be the case with aluminium, and sometimes with manganese.

This brings us to the question of the alloys of iron, of which, with regard to magnetism, very little is yet known. A research of them, comprising their history, their chemical constitution and their physical and mechanical properties, would undoubtedly lead to very interesting results. We should find alloys, which have no characteristic feature of their own, but simply show the magnetic properties of the iron, rapidly decreasing with increasing percentage of alloying material, as it seems to be the case with the iron alloys of quicksilver, aluminium, etc. Perhaps cast-iron, as carbon alloy, ranges here.

Other alloys are characteristic bodies, magnetically different from either of their constituents, as the nickel and manganese alloys. In other alloys, again, a very small percentage of alloying material has a great influence upon the magnetic constants, not directly, but indirectly, by the chemical and physical changes brought about by the addition of the alloying material to the fused iron, though this material may no longer exist in the finished iron, but have passed into the slag. So a very small percentage of aluminium and even of manganese or titanium may improve the mechanical and magnetic qualities of the iron by reducing the oxide of iron dissolved in the fused metal and causing the separation of carbon as graphite, becoming oxidized thereby itself. In this case, with increasing percentage of alloying material, the action reverses, and while a small percentage of manganese added to the fused iron increases its permeability, a larger percentage rapidly decreases it. This is the case where the *history* of the iron is of main importance, since chemical analysis does not record the added aluminium or manganese which has passed out by oxidation. But all these problems are still unsolved, offering a large and promising field for further investigation.

Yonkers, N. Y., Sept. 10th, 1892.

NOTE.—In part I. of the paper "On the Law of Hysteresis," of Jan. 19th, 1892, on page 51, in the second column of Table XIV. it should read:

$B = 14.500$ $B^{1.6} = 4.552$
16,000 5.329

C. P. S.

DISCUSSION.

THE CHAIRMAN [Vice-President Hammer]:—It may be desirable at this hour to postpone the general discussion of a paper of such magnitude and importance until a future meeting. But there are among our members to-night some gentlemen who have given this class of work their particular attention and it would be interesting to hear from them. Most of them, I believe, have received advance copies of the paper. We should be very glad to have Mr. Kennelly open the discussion.

MR. A. E. KENNELLY:—I think it will be unnecessary for me to express the general and very high opinion in which we hold the paper we have just listened to. It is a classic to us and I think it will be a classic to a great many more than ourselves. The Institute may well congratulate itself upon this paper having been read before it.

Let us, in a few words, try to outline some of the facts which we learn here for the first time. About two years ago Mr. Steinmetz first drew attention to the fact, then unnoticed, that when you magnetize a piece of iron between a certain terminal negative value and a corresponding terminal positive value—say, 5.000 c.g.s. lines per sq. cm. in one direction, and 5,000 in the other direction—the area of the enclosed Ewing loop or the hysteretic energy which had been given to the iron was a certain definite function of the maximum magnetization, namely, it varied as $B^{\frac{8}{5}}$ where B was the maximum value. That in itself was a discovery, but it was found to agree with results which had already been obtained. Ewing's own curve supplied that law. But no one would have supposed, at first sight, that if you took a piece of iron and magnetized it from 5,000 lines positive to zero and back, or from 5,000 lines positive to 2.000 lines positive and back, that you would still have the same law within that limited range. Mr. Steinmetz has shown us that it does follow even in that case. The loop itself is not the same. But the new loop, under those conditions, still retains between these values the law of the $\frac{8}{5}$th power, and I heartily congratulate him upon that discovery.

Besides that, which would be enough to immortalize one paper, we have a very interesting method of measurement given to us, upon which, perhaps, the attainment of these results depend; for it may be that without that means of measurement, Mr. Steinmetz would have had much greater difficulty in arriving at the numerous results there given to us than he actually has had. The method consists in putting, as he has described, a second coil upon the tested iron and connecting this independent coil to the wattmeter. As shown in the paper, I take it, the diagram in Fig. 1 is diagrammatical only, because there would be under the conditions of winding there indicated, a considerable magnetic leakage, and I presume, from the paper, that the three coils were wound on together. It is well to point that out, because any one trying to repeat that measurement might, by following the diagram, be led into difficulty.

With respect to the units that Mr. Steinmetz employs, I am sorry that I have the honor to differ from him. I think we should discuss this matter freely, because when a paper of this importance is generally circulated it is very desirable that we should know just in what units the results are expressed. There is nothing more convenient, one must acknowledge, than taking the total number of ampere-turns on a magnetic circuit as the magneto-motive force. There is a simplicity about it that recommends itself. But, unfortunately, it is very doubtful if you can confine that simplicity to the case which it is intended for. For example, the reluctivity that Mr. Steinmetz employs is $\frac{1}{1.257}$ times the reluctivity in the ordinary units, and the magneto-motive force is reduced to suit. The notion is very plausible that reluctivity could be kept as a thing apart in the electro-magnetic units, and that so reserved, it would not alter the general system of absolute units by a deviation from the ordinary rule. But there is this danger with magneto-motive force, particularly in the sense in which Mr. Steinmetz employs it, namely, that magneto-motive force per centimetre of circuit is really of the nature of a flux density, that is to say, so many lines of force per sq. cm. When you divide the total magneto-motive force in a circuit by the total length of that circuit you get a quantity of the nature of intensity or lines of force to the unit of area, and there will always be the danger of persons confusing the entire electro-magnetic system by using this unit. For example, in describing the intensity of the earth's field, you might get drawn into using a value for the earth's intensity of field which would be 20 per cent. too small, and which would conflict with all our existing notions. The existing system of electro-magnetic units may perhaps be likened to a card-house. It contains defects, but if you try to pick out the defects by taking out one card, you may destroy the whole construction. So that while one cannot but admire the simplicity of this device, there is a danger in it. However, all that we have to do in this case is to add 25 per cent. to the stated reluctivities and to the magneto-motive forces in order to convert them into the ordinary values.

Mention is made in the paper about iron being improved by aluminium up to a certain point, and afterwards the reverse action taking place. I think the fact of the improvement consists in the absence of the aluminium. That is to say, up to a certain point you put aluminium into the material that is going to be cast, and in the process of casting that aluminium disappears in combination with other impurities gaseously. The resulting iron is left more nearly pure. But if you put too much aluminium in, a certain excess will remain which is a new impurity and so affects the curve.

Mr. Wm. Stanley, Jr.:—It seems to me that Mr. Steinmetz has done for the magnetic circuit very much what Ohm did for the electric circuit. He has defined the law relating loss of energy to flux. I feel utterly unable to discuss the paper in the same terms Mr. Steinmetz has given it to us, because few of us have been equipped with the knowledge and the facilities necessary to investigate the problem as he has. But to the constructing engineer working with the alternating current appliances of to-day, the paper of Mr. Steinmetz affords more assistance than anything we have ever had the pleasure of listening to.

One of the most remarkable things about the paper is the agreement of the results. We are accustomed to look upon decimal figures of the third place as rather uninteresting and are skeptical as to their value, but Mr. Steinmetz's results seem to show the most remarkable agreement.

Can Mr. Steinmetz give us any physical picture that will allow us to realize in any way how it is that this wonderful discovery that he has made is true—how it is that when the induction is varied between any two limits, the loss of energy is the same? If we consider that we have 2,000 lines passing through a centimetre of iron, and add 10,000 more to it, we seem to use up the capacity of that centimetre, and it seems natural that the energy spent must be greater than if we reverse the magnetization between equal limits through the zero point of magnetization. Can Mr. Steinmetz give us any picture of how it is that the hysteretic loss due to the changing magnetization is constant when the included limits of induction are the same?

Mr. Steinmetz has spoken of the change of the magnetic hardness of the steel that he has experimented with, and the effect of the alternating current upon that property. I am not prepared to say now that we have discovered that iron, subject to alternating magnetization, ages, but we are very suspicious of it. We have found that transformers whose hysteretic loss was well known at the time they were manufactured, after being in continuous service for over a year had an increased hysteresis in some cases amounting to 40 per cent., and we found, after pulling out the cores of the transformers and rebuilding them (placing them in other coils and rebuilding them), that we were unable to change the hysteretic loss unless we *re-annealed the iron*. When the transformers were first made, the iron was very carefully annealed and it was extremely soft. The iron which we employ, when up to its standard value, has a hysteretic coefficient which corresponds, as nearly as one can reckon, with the coefficient given by Ewing. It is American iron, made especially for our purposes, almost free from carbon and is extremely uniform. But the slightest alteration of the annealing condition produces enormous differences in the quality of the metal. For example, the metal is obtained in sheets which are approximately 4 feet long and 2 feet wide. The iron at the edges of the sheet is well annealed—blued, extremely soft, and its hysteretic coefficient is very low. The iron taken from the centre is often very different—much harder, and may have a hysteretic coefficient 20 or 30 or, possibly, 40 per cent. higher, and, furthermore, the entire sheet may be made uniform by having it cut up and properly annealed; so that it is very necessary to carefully anneal iron in all cases where it is used in a magnetic core.

Mr. Steinmetz speaks of ferrotype iron. I have found it to vary greatly, and I have found it was the most difficult metal I have ever attempted to get into uniform and standard shape. But if the iron be very carefully annealed and if the process be continued for a week; if the iron be heated up to a red temperature and then be allowed to cool very slowly for a week, it possesses a very low hysteretic coefficient, with extremely high permeability. I was greatly interested in the description Mr. Steinmetz gave of the loss of energy, in the case of the iron filing experiment, being much greater than could possibly be due to hysteretic loss, and it occurred to me to ask him how the iron filings were placed in the field—whether it was possible that the field shifted. If one takes a test tube partly filled with filings and places it in a moving or Tesla field, the filings will be seen to jump around and, if the field be made strong enough and its direction shift enough, the filings may be pulled in various directions inside the tube.

I cannot pass by the opportunity of asking some of the members here to criticise an experiment we have made. The experiment is this [making a sketch on the blackboard] and it is probably misleading, but it has not been explained. If a coil of wire be wound around an iron core, and especially a core which has an air-gap which does not form a closed magnetic circuit, the magnetizing power required to magnetize the core will be dependent primarily on the potential which is employed and upon the reluctance of the circuit. An alternating E. M. F. being applied, a certain amount of current will flow dependent upon the reluctance. Now, that component of the current which does flow and which lags 90 degrees behind the energy current can be supplied by a condenser located in parallel with the coil. Such a condenser is supposed to be represented here. So that if the coil required 10 amperes of current to magnetize it, and the energy wasted is represented by one ampere, then, theoretically, we ought to be able to supply the one ampere from the source of supply, while the lagging current would come from the condenser and would be equal to the $\sqrt{10^2 - 1^2}$. These conditions, however, are practically impossible, because of the fact that the hysteretic loss in the iron distorts the shape of the wave of current and it is no longer a sine-shaped wave. We therefore, instead of being able to supply one ampere of current from the source of supply, have to furnish a current of about three amperes, or one-third of the total value. Now, if, instead of using a frequency of 130 periods a second, in this experiment we decrease the frequency to one-half, keeping the magnetization the same, and if we again place a suitable condenser in parallel, we will be able to furnish more of the current from the condenser than in the first place. Now, why is it that we cannot furnish the entire current from the condenser? Obviously because, in the first place, of the shape of the wave, and secondly, I think, because the whole wave lags in time. With a sufficiently low frequency we are able to almost entirely supply the necessary current from the condenser, the source of supply giving only the energy current.

Now, I am aware that this is rather away from the subject in question, but if the theory that I hold of it is correct, the wave of magnetism in the coil, does not lag at low frequencies as much as it does at the higher frequency, while the loss of energy is less at the lower frequency; it is so small in all cases as to be inconsiderable. The experiment has never been tried with sufficient accuracy to warrant my giving any more than this suggestion of it, and I would like it to be pulled to pieces by Mr. Steinmetz and some of the other gentlemen, if they can do it.

Mr. Mailloux:—I would like to ask Mr. Steinmetz about the formula on page 679 ($H_{\infty} = \eta L_{\infty}^{1.6}$). I do not quite see how one can get infinity to a higher power.

Dr. Chas. E. Emery:—This paper of Mr. Steinmetz has evidently required an enormous amount of earnest work. The results in general are novel and some of the experiments so exhaustive, that further investigation in the same direction seems unnecessary. While we expect to criticize some of his generalizations, his demonstration that what he terms "loss by hysteresis" is proportional to the "amplitude of magnetic fluctuation" independent of absolute values, is a very notable example of successful experimental investigation, for which, as well as the clear and complete manner in which the subject has been examined and presented, he is to be commended and congratulated.

Years ago when making original investigations in an entirely different field, I found it desirable at times to stop and *think*, and especially to compare the bearing of the recent work upon that previously accomplished, and endeavor to make some practical application of the digested results. If we adopt this policy in relation to this paper, and calculate the so-called loss by hysteresis in the core of an armature making only 1,000 revolutions per minute, with the magnetization alternating between minus and plus 16,000 lines per square centimetre, we find that according to the formula of Steinmetz on page 685 of his paper such loss will be something over 13 per cent. We all know that such a loss is impossible. Dynamos and motors show mechanical efficiencies of over 90 per cent., and they as well as transformers show electrical efficiencies so nearly 100 per cent., that when the losses due to resistances are considered there is no loss of practical value left for hysteresis or even eddy resistances. Prof. Ewing (page 315) notes from experiments of Mordey, that the apparent loss by hysteresis in the core of an armature is not as great as shown by calculation, and states "the molecular theory makes it probable that the work spent in reversing magnetism will be less when the reversal is accomplished by rotation in a constant field, than when it is accomplished by reducing the magnetic force to zero and restoring it with sign reversed." This explanation, even if sufficient, does not explain how transformers can be so efficient if there be a loss by hysteresis even of a few per cent. In the other cases, however, it does not seem possible, even considering all reactions between field and armature, that the magnetization is not in all cases reversed in the latter. Now, however, Steinmetz shows that it makes no difference even if such reversal does not take place. The apparent loss is due to the total amplitude of change, not to change of sign. This additional evidence makes it proper to assert, as has been my impression for a long time previously, that the loss by hysteresis is not proportioned to the reduction of the magnetization by pulsating and alternating currents, but that the phenomenon of hysteresis merely shows a reduction in the inductive capacity of iron; that the change of magnetic potential, merely operates to change the permeability, and though the curve of magnetization is reduced by the area of the loop showing the effects of hysteresis, this does not produce loss of energy in any greater sense than is involved in the difference of permeability for different degrees of exciting force under other circumstances. In my recent paper on "Magnetization", magnetic phenomena are examined on the basis that magnetism is due to etheric flow which is intensified by the action of molecular magnets. The phenomenon of hysteresis can only affect this intensification, or what is called by Ewing the "metallic induction", and on the basis stated we can well explain hysteresis on the principles of inertia and delayed action between cause and effect, knowing that the molecular magnets are masses, even though minute; that they are also subject to molecular constraint, and consequently that there is such a thing as "time lag" in magnetism (page 322 of Ewing), so that the result is exemplified very familiarly by the delayed action of the tides.

It may be claimed that loss must ensue on account of the resistance, or the "reluctance" (if the majority wish so to call it), of a magnetic circuit, and that energy must be absorbed in overcoming such resistance. This is true, but it is provided for by the extra turns which produce additional exciting force sufficient to overcome the resistance. If the reluctance is increased by changes in the magnetic potential, the number of lines that can flow are reduced, the same as if the air-gaps in the circuit were increased. The energy represented by the extra exciting force is a loss which must appear as heat, but such heat is not due to hysteresis in any more direct sense than an increase of reluctance caused by air-gaps or by crowding more lines through a given area. It seems evident, therefore, that the loss by hysteresis is to be measured not by the loss in magnetization but by the proportional increase in exciting force necesssary to overcome the increased reluctance, exactly as losses are now measured for reluctances due to other causes. It is thought best to group all the causes which decrease the permeability, or in other words increase the reluctance, in one group, when it will be found, as has been the case with both dynamos and transformers, that it is most economical to increase the weight of the iron, of course within reasonable limits, and thus reduce the intensity of magnetization and secure the advantage of the great increase in permeability due to forcing fewer lines through unit area.

The generalizations in the paper by which the magnetic properties of different materials are expressed as constants in various equations are very interesting, and will undoubtedly serve valuable purposes. It cannot, however, be allowed that one application of the Frölich function is sufficient to express through wide ranges the relation of the exciting force to the magnetization. Ewing (page 257) designates the reciprocal of the permeability by the character ρ and calls it "the specific magnetic resistance." To this quantity Kennelly applied the term "reluctance" (crediting it originally to Heaviside), and derived its value from one of the forms of the Frölich function, calling particular attention to the fact that it showed a linear relation between the reluctance and the exciting force. Steinmetz, however, adopts the original application of this function to the *metallic* induction as given by Ewing (page 320). Kennelly found for the experiments examined by him that the equation for the reluctance required two applications to approximately represent the experimental relations, whereas Steinmetz appears to claim that if the metallic induction be used instead of the total induction, the relation can be satisfactorily represented through satisfactory limits by one application. The function in all cases is that of Frölich, and the fact that it is stated in different terms makes no difference. The result finally, when the relations of B and H are plotted in a curve, must be the same as if the Frölich function in its original form were employed to compute points in such curve. S. P. Thompson in his work calls attention to the fact that the Frölich function is not satisfactory in all cases, and in Part II of my recent paper on "Magnetization," Fig. 3 shows the application of this function, with Mr. Kennelly's constants, to the experiments from which such constants were derived. The result is, that the calculated and experimental curves coincide at two assumed points (independent of the origin from which all curves of this kind must start), and approximately near such points, but for the higher values the curves are rapidly separating. The adoption by Steinmetz of Ewing's application of the Frölich function to the metallic induction instead of the total induction used by Kennelly is quite insufficient to make the function applicable to a much greater extent to different materials, for the reason that within ordinary practical limits the metallic induction forms a very large proportion of the total induction, as may be seen readily by referring to Fig. 6 in my recent paper previously referred to, where the metallic induction designated *L* by Steinmetz is called *I*, following the lead of Ewing (page 320) where he states the form of the Frölich function. This, however, should have been designated by I_1; or some other distinguishing character, as it is not the same *I* used by Ewing in the earlier chapters of the same work. Both refer to the metallic induction, but *I* is first used by Ewing to express the "intensity of magnetization," or the induction, in turns of current in absolute units, whereas it is employed the second time, and so used by myself, to express the metallic induction or concentrating influence of the iron in magnetic lines. In writing Part II of my paper on "Magnetization," I endeavored to bring together the latest information as to the relation of magneto-motive forces and the resulting magnetizations, so as to give empirical formulae showing with satisfactory accuracy such relations. Among other things I selected a most complete table published by Mr. Steinmetz in Part I of the paper on hysteresis, now under discussion, relating to experiments with sheet-iron and agreeing well with two accompanying tables referring to the same material, but when the results had been tabulated in connection with other experiments, I was obliged to state (Sec. 28) that the initial results were so low and the others so high, that it was necessary to hesitate in accepting them without further investigation. In the present paper, Part II, Mr. Steinmetz gives a similar table showing the average magnetic properties of five samples of sheet-iron, obtained, so far as can be gathered from the paper, by the use of the same experimental methods, but the higher values are 18 per cent. less than those given in the first table, and others accompanying for the same class of material. Several values taken from the two tables are given in parallel columns in the accompanying table with corresponding data from other sources; for instance, results given by S. P.

MAGNETIZATION OF SHEET-IRON.

Exciting Forces.		Steinmetz On Hysteresis.		Hopkinson. Thompson.	No. 8 Cornell.
H_1 or F	H	Paper I. B	Paper II. B	(Wrought Iron) B	B
5	6.3	9200	7706	10600	10150
10	12.6	13070	11723	13300	13100
20	25.1	15200	13975	14750	14450
40	50.3	17050	15525	15950	15500
60	75.4	18650	16135	16600	16100
80	100.6	20080	16480 calc.	………	
83.5	105	20300	16420	17000	
278.4	350	………	17350	19000	

Thompson with wrought-iron, and one of those recently obtained at Cornell University with sheet iron and referred to specifically in Part II of my paper as the "Cornell experiments." The comparison shows distinctly that the higher results first given by Mr. Steinmetz were altogether too high. The intermediate results from the later table correspond more closely to those given by others, but the initial ones are still very low, and if we calculate the higher values with the constants given, we find that these are also low compared with those given by Thompson based on Hopkinson's experiments. It may be that Mr. Steinmetz is right and others are wrong, but the discrepancies pointed out are sufficient to make us fear that in undertaking work of such magnitude, errors due to calibration of instruments, and those due to particular methods, have crept in so that we cannot be certain of absolute results at all limits, though the value of the comparative results for different materials cannot be questioned. The application of the Frölich function in Fig. 3 of my paper shows that it tends to reduce the higher values, and in applying the same in a foot note, Sec. 32, I was obliged to apply the function twice to approximate the experimental results. With our present information, therefore, we must claim that the Frölich function, even in the form used by Mr. Steinmetz, is not always applicable, and the generalizations based thereon cannot as a whole be accepted, though the tabulated values are undoubtedly of great value for comparison with other results obtained with the same apparatus, and with instruments standardized on the same basis.

MR. TOWNSEND WOLCOTT:—I think one of the most interesting results of Mr. Steinmetz's investigation is that the loss of energy for which he has constructed his formula is not necessarily hysteresis loss at all, but the molecular friction loss. That is to say, there is no necessary relation between the hysteretic loop and the loss by alternating polarity in the iron. A number of years ago, before the different permeabilities of iron and air were so thoroughly investigated, there were a number of inventors devoting their attention to making machines without any iron in the armature. Among them was Dr. Chas. A Seeley. I had some business connections with him. We thought at that time that there was, in addition to the loss in the iron by Foucault currents, some sort of loss by the reversal of polarity. We had no very definite ideas on the subject, but so far as our ideas did go, we considered that the loss was due simply to the change of polarity of the iron. In other words, the changing of the value of induction. As far as we knew, there was no necessary connection between that and any phenomena of lag like hysteresis. Later on, we learned from Ewing that the energy dissipated by hysteresis is represented by the area of the loop. Now, Mr. Steinmetz proves that that is an error, and we come back to the original condition of affairs—that the energy dissipated by magnetization of the iron is not, strictly speaking, energy dissipated by hysteresis. There is no necessary relation between the two. That is, the hysteresis can be abolished by vibration, and still the dissipation of energy is just the same.

Mr. Steinmetz:—Mr. Kennelly's supposition with regard to Fig. 1 is right. This figure is diagrammatical only, and therefore shows the magnetic circuit as a ring with separate exciting and exploring coil. In reality, certainly both coils were wound together with the same number of turns, both wires wound simultaneously to avoid magnetic leakage between them, as explained in the paper.

With regard to the second point which Mr. Kennelly mentions, I entirely agree with him that our present system of units, though certainly having many defects, is still the best we can devise at present, and I did not intend to introduce a different system of units. But these researches had been undertaken, first, for a strictly practical purpose, and there, ampere-turns are the units derived from the tests; ampere-turns are the units used for designing electric machines, and hence the circuitous over "field intensity" H is unnecessary. The experiments have since developed into scientific research, and in publishing them I hesitated whether I should not reduce the whole to absolute units H. But then I came to the conclusion that the original results are necessarily more exact than the derived values and, besides, the time failed me for the reduction of some thousand readings. So I simply drew attention repeatedly to the units used, giving the reductional factor between them, and while retaining the customary symbols H, μ, κ for the established units, I introduced the symbol F for the "ampere-turn per unit length of magnetic circuit" as an auxiliary unit. "Reluctivity" not yet having a symbol of its own, I saw no objection in using ρ for the function $\frac{F}{L}$, believing that anybody who reads a paper of some hundred pages through should avoid mistaking symbols F and H, while for practical purposes the use of F may be very often convenient.

With regard to aluminium, I noticed the same point referred to by Mr. Kennelly, for a qualitative analysis of the mitis metal revealed no perceptible trace of aluminium, and this very fact brought me to the opinion expressed in the appendix, that the aluminium improves the magnetic quality of the iron only in-so-far as it acts as de-oxidizer of the oxides of iron dissolved in the fused metal, in a similar way as titanium or manganese, but passes out again in the slags, while when remaining in the iron, it spoils it magnetically.

With regard to the decimals given in the figures, I have, with few evident exceptions, followed the rule of astronomical calculation—to give one decimal more than can be relied upon, to make the figures fit for further calculation—so that the last decimal is within the errors of observation, the forelast usually correct.

I am sorry not to be able to give Mr. Stanley a picture showing how it is that the area of the hysteretic loop is independent of the absolute values of the limits, and can only say that I did not expect this result at all, and was very much surprised as I first noticed in the electro-dynamometer tests the voltmeter and wattmeter readings remaining in a constant ratio, independently of the ammeter reading. I can only think that induction and hysteresis must depend somehow or other upon the same law and thereby show this constant relation.

With regard to the increase of hysteresis in soft iron by "aging," I never had occasion to observe this phenomenon, but I think that it may be due to incipient crystallization caused by the constant and prolonged molecular vibration, which may increase the magnetic hardness and hysteresis. It would be highly interesting to see whether in such a case the magnetic characteristic has changed also and a has increased.

In the tests made with iron filings, I believe a shifting of the field was excluded. It certainly was in the magnetometer tests, and they show the largest increase of coefficient η.

With regard to Mr. Stanley's experiment with condenser in shunt to open circuit inductor, I can not believe that the difference between high frequency and low frequency, is due to a change in the phase of the inductor current. For if the current is the same, and eddy-currents excluded, the hysteretic loop, and consequently the wave-shape and the phase of the inductor current, are the same for all frequencies. But perhaps the dielectric hysteresis of the condenser—which, though small, is not negligible compared with the small hysteretic loss of an open circuit inductor—has something to do with the phenomenon. The dielectric loss in the condenser seems to be proportional to the square of E. M. F. and square of frequency. But, for a given current, the E. M. F. is inversely proportional to the frequency. Consequently, for a given current, the dielectric hysteresis of the condenser is constant for all frequencies, while the hysteretic loss in the inductor—for a given current—is proportional to the frequency. This may have something to do with the cause of this phenomenon.

With regard to L_∞, Mr. Mailloux has misunderstood the symbol. L_∞ is not infinite at all, but ∞ merely an index, and L_∞, as explained in the paper, denotes that very finite numerical value which L approaches for infinitely increasing M. M. F.'s, F. Hence $H_\infty = \eta \, L_\infty^{1.6}$ is not infinity to a higher power, but entirely finite.

Now a few words on the remarks of Dr. Emery. First, I am highly astonished to see him give a loss of over thirteen per cent. in the armature core of a dynamo, as calculated from my tests. I am inclined to think that, due to the short time left between the distribution of the advance copies and the reading of the paper, an error must have crept into his calculations. I have made many calculations of armature losses, for all sizes of machines, and have almost always found the hysteretic loss amount to a *fraction of one per cent.*, so that I am almost inclined to think that Dr. Emery has mistaken the number of revolutions per *minute* for the number of cycles per *second*, which would bring his figures down to one-sixtieth, or about one-quarter of a per cent., a value found in practical dynamo machines. Here I must correct a mistake also. The hysteretic and eddy-current losses in *dynamo machines* do *not* enter into the *electrical* efficiency at all, but only into the *mechanical* efficiency, there showing as an *apparent* increase of the *mechanical friction*.

With regard to transformers, their design has reached now a development where the iron losses are not only calculated by the law of 1.6th power, but found by experiment to agree with the calculation, so that there can be no more doubt about their constancy, and they amount in the Ganz and Company transformers (42 periods, $B \sim 5{,}000$) to 2½ to 5½ per cent.; in the Siemens-Halske transformers, to 2 to 6 per cent. (according to the size); in the new Stanley transformer (17,500 watts, 133 periods—Cornell University tests), to .9 per cent.[1]—so that Dr. Emery will see that this hysteretic loss does not exclude the high efficiency of the transformers.

There are, however, now quite a number of efficiency tests of transformers published, where the losses are separated so that the hysteretic loss can be seen to agree with the formula and still to agree with as high an efficiency as 97 per cent. in the Stanley transformer.

With regard to the notion that the hysteretic loop merely represents a variation of the reluctance but no loss of energy, I can be short, because it has been proved by Warburg, and by Ewing, that if a magnetic circuit undergoes a cyclic variation, an amount of energy disappears out of the M. M. F., which is equal to the area of the hysteretic loop, and consequently if neither external work is applied to, nor work done by or in the magnetic circuit, from the *law of conservation of energy* follows that this energy is consumed in the iron by what we call molecular friction. We can not get over this well-established fact. With regard to the correspondence between induction and M. M. F., the hysteresis indeed appears as a cyclic variation of reluctivity, but it represents nevertheless a consumption of energy equal to the area of its loop.

The "time lag" in magnetism, observed by Ewing, is an entirely different phenomenon from what has been called "viscous hysteresis." This time lag takes place after seconds, and even minutes, and consequently it has nothing whatever to do with the inertia of the molecules, which does not show up noticeably at frequencies of over 200 complete periods per second, neither can it be expected.

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[1] The smallest loss I ever saw in transformers.

With regard to the extra turns, no such thing exists in the modern transformer, but the ratio of the turns is the ratio of transformation.[1] They may have been necessary in some older types with extreme magnetic leakage.

The magnetic resistance, or "reluctance," does not consume any energy, as Dr. Emery seems to think, like the electric resistance, but a magnetic circuit can remain constant for any length of time without expenditure of energy therein, as is well-known, for otherwise the permanent magnet would represent a perpetual source of energy.

Consequently, the loss by hysteresis can not be measured by the proportional increase in exciting force, nor by the loss of magnetization, as Dr. Emery misinterprets my explanation, but it was proved long ago to be identical to the area of the hysteretic loop, by the law of conservation of energy, which, I believe, stands beyond doubt.

With regard to Fröhlich's function, or the linear law of reluctivity, Dr. Emery makes a mistake by saying that Mr. Kennelly applied the name "reluctance" to the reciprocal value of the permeability, and called attention to the fact that it shows a linear relation to the M. M. F., while I applied this linear law to the metallic reluctance. I must decline this honor, for Mr. Kennelly has expressly and clearly stated, in his classic paper,[2] that he applies the linear law to the "*metallic reluctivity*," but *not* to the *inverse value of permeability*, and has brought such ample proof for the agreement of this linear function with the tests that I did not think it worth while to give the experimental proofs for it, but thought it sufficient to simply state the fact of the agreement of my tests with the linear law.

Fröhlich's function, though very satisfactory within a limited range, had to be abandoned, because it did not hold for high values of M. M. F., and it was Kennelly's merit to prove that by substituting for "*induction*" the term "*metallic induction*," the hyperbola laid through any two points—beyond a minimum value of M. M. F.—of the *metallic* magnetic characteristic, *coincides with the whole characteristic within the errors of observation* and *does* NOT *separate rapidly for the higher values*, as Dr. Emery says, and as indeed the whole induction B does, and I found the same.

Neither has Mr. Kennelly made two applications of Fröhlich's formula, nor did I intend to express the whole function by one hyperbola, but Mr. Kennelly has tried to express the reluctivity at the initial inwards bend by the function $\rho = a - b\,H$; I, by the addition of the term $\gamma e^{-\delta F}$, but not with quite satisfactory result in either case.

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[1] Ganz and Company 7,500 watt transformers :

Ratio of turns, $\frac{1080}{60} = 18.$

Terminal pressure, $e = 1,929$ volts ; $e' = 105$ volts.
Consumed by resistance, $4.2 \times 4.28 = 25.7$ volts ; $.013 \times 75 = 1$ volt.

Hence, ratio of E. M. F.'s, $\frac{1900.3}{106} = 17.93.$

[2] TRANSACTIONS, vol. viii., p. 485.

With regard to Dr. Emery's remark, that he has, in his paper, "applied Mr. Kennelly's constants to the experiments from which such constants were derived," but had found them to disagree with the tests, Dr. Emery is mistaken, for he did *not* apply the constants to the curve from which they were derived, since Kennelly expressly stated that his linear law of reluctivity applies to the *metallic induction*, which reaches a finite value of saturation, as Ewing proved, and Dr. Emery consequently could not expect to see it agree with the whole induction, which continues to rise infinitely, as Ewing proved also.

With regard to the symbol I, Dr. Emery's quotation from Ewing, page 320, is wrong, for neither there nor anywhere else does Ewing use I for "metallic induction," but consistently applies this symbol to "intensity of magnetization," the same as I and everybody else did, so that there is no need for a distinguishing index for Ewing's I.

The magnetic characteristic of sheet-iron, in my present paper, which Dr. Emery refers to, is *not* the average of five samples, as he says, but derived from one sample only, and one of less than average permeability, as the value $L_\infty = 17.24$ proves.

Now, with regard to the values given in Dr. Emery's table. In the highest values taken from my first paper, the M. M. F. is given by far too low, as explained in the appendix to my present paper. These values were derived by the electro-dynamometer method, specially explained in my former paper, and in this method the M. M. F. calculated from the effective ammeter reading comes out too low for high saturation, due to the discrepancy of the current wave from sine-shape, as will be seen by plotting the curve. This was the reason why, in the present paper, where I laid more stress upon the determination of the magnetic characteristic, I abandoned the dynamometer method for reluctivity determinations at higher saturations altogether, and used the magnetometer method, after having determined its reliability at these saturations by comparative tests with ballistic galvanometer.

That the numerical values of B for different observers differ, I can not help, since different samples of iron differ quite considerably. For instance, Ewing (page 107) finds for a very soft iron wire for $H = 75.2$ only $B = 15{,}560$, much less than any of the figures Dr. Emery refers to, while the tests given in my present paper show values of absolute saturation from $L_\infty = 15{,}750$ up to $L_\infty = 20{,}100$. The reason of these discrepancies is simply that different kinds of iron differ considerably in their magnetic qualities. For lower M. M. F.'s especially, Dr. Emery will find this still more noticeable. For $H = 10$, for instance, the Norway iron in my tests gives $B = 13{,}300$, even more than the value Hopkinson gives.

For lower M. M. F's, however, the values found by the method of reversals, which I exclusively use because it gives results not influenced by the remanent magnetism of former tests, are always lower than by the step-by-step method (*cf.* Appendix I).

In consequence hereof, I can not concede that I have made any mistakes in the calibration of my instruments, but I rather prefer to stick to my former opinion, that different kinds of iron have somewhat different magnetic characteristics; the more, as quite a number of such different characteristics are given in my paper.

Coming, now, to the conclusion, I can only say that as far as our present knowledge goes, the linear function $\rho = a + \sigma F$ expresses the *metallic* reluctivity correctly within the errors of observation, without requiring a repeated application; and that, since Dr. Emery was not able to bring any experimental proof whatever for his assertion, that the linear law does not apply to the *metallic* induction, after the complete and classic researches of Kennelly, which I found corroborated by my own experiments, I am sorry to disagree with Dr. Emery's assertion.

Some smaller mistakes which crept into his critical remarks, I may be allowed to pass unnoticed here.

THE CHAIRMAN:—[Vice-President Hammer]. We will now hear the report of the Committee on Revision of the Rules, respecting the election of officers.

MR. T. C. MARTIN:—The report is in the hands of the Institute and accepted, and referred back to this meeting from the meeting at Chicago. The Committee has nothing to do with it at the present time, except to vote in support of the recommendation.

THE CHAIRMAN:—I believe all the members have received copies of the Report of the Committee. I think it is scarcely necessary to read that report, unless specially requested. [See page 460.]

THE SECRETARY:—It was the sense of the meeting at Chicago that the final discussion of this matter should take place at the next regular meeting in New York, and the Secretary was instructed to have the report printed and circulated and this has been done.

MR. PHELPS:—I move that the report of the Committee be adopted. [Seconded.]

THE CHAIRMAN:—It is moved that the report as presented by this Committee be adopted.

[The motion was carried.]

MR. PHELPS:—That makes the report a part of the law of the Institute.

THE CHAIRMAN:—Yes sir. In view of the lateness of the hour it may be desirable to adjourn, but if the members desire that Mr. Steinmetz should proceed with his remarks, he may do so.

[On motion the meeting adjourned.]

APPENDIX II.

1. *Efficiency of Electro-Magnetic Conversion of Energy.*—Since now the loss of energy by molecular friction in the iron is found to be proportional to the 1.6th power of the magnetic induction,

$$H = \eta\, L^{1.6},$$

while the energy converted from electric to magnetic energy, and vice versa, is known to be expressed by the equation

$$W = \int_{B_1}^{B_2} F\, d\, B,$$

we are enabled to calculate the *efficiency* of this electro-magnetic conversion of energy.

The *transfer of energy* during a complete magnetic cycle between the limits $+L$ and $-L$ is approximately

$$4\ W = 4 \int_{0}^{L} F\, d\, L$$

(so far as the transfer takes place by the *metallic* induction).

The *loss of energy is*

$$H = \eta\, L^{1.6};$$

consequently the *efficiency of the electro-magnetic conversion of energy*

$$\delta = \frac{4\ W - H}{4\ W}.$$

Such an efficiency curve, referring to the sheet-iron in Chapter I., is given in Fig. 30.

This electro-magnetic efficiency has a very important bearing upon the storage of energy by magnetic potential, as it is frequently made use of for the shifting of phase of alternating currents, for it determines the unavoidable loss encountered in such a storage. For in the case represented by Fig. 30, of the amount of electric energy stored by magnetic potential in the range of medium induction, not more than about 88 per cent. can be derived back as electric energy. Consequently, shifting of phase of alternating currents by means of magnetic potential must be rather uneconomical. It must be understood, however, that the curve in Fig. 30 refers only to the one sample of sheet-iron.

2. *Limits of the Law of* 1.6*th Power.*—I have shown that the law of the 1.6th power holds within the errors of observation for the whole range of magnetic induction from $B = 85$ lines of magnetic force up to over 19,000 lines per cm.^{2} Outside of this wide range the law has not yet been tested, and though it is not

likely that for very high saturations an exemption will take place 19,000 being already very near to absolute saturation, the case $B < 85$, that is, extremely low magnetization, requires a further investigation.

The loss of energy per cycle is

$$H = \eta B^{1.6}. \qquad (1)$$

The energy derived per cycle from the M. M. F. is

$$2 W = 2 \int_0^F F \, d B. \qquad (2)$$

Now, evidently the value H can not be larger than $2 W$, that is, not more energy can be lost than is available, and if there exists a point where (1) becomes equal to (2), below this point one of the two equations, (1) and (2), must cease to exist, and since (2) is based upon the law of conservation of energy, it can only be (1), that is, at the point where for extremely low M. M. F.'s (1) becomes larger than (2), there we must expect a limit for the empirical law of the 1.6th power.

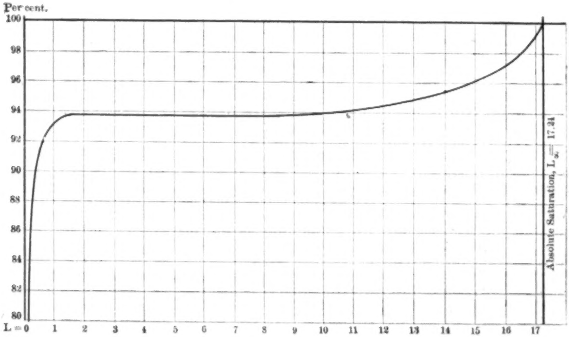

Fig. 30.

This really seems to be the case, for the lower we come down in the value of F, the fuller and "fatter" the hysteretic loop becomes.

For extremely low M. M. F.'s, the reluctivity ρ seems to approach a finite limiting value ρ_0 (Rayleigh).

Then $B = \frac{F}{\rho_0}$. Consequently we get (approximately),

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